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Effect of multiple equilibria on power system stability

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Abstract

This paper illustrates that for the same system loading, a power system may become unstable either due to angle instability or due to voltage instability or due to both depending upon the disturbance. The studies on two sample power systems demonstrate that for certain range of system loading there are many controlling unstable equilibrium points and their unstable manifolds give rise to different kinds of the instability for the same loading. The stability region in parameter space has also been characterized considering saddle-node and Hopf bifurcation to illustrate that the Hopf bifurcation further reduces the stable range of the operating parameters. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Non-linear dynamical system theory successfully explains some of the most complex behavior of power systems. References [1] and [2] contain a survey of bifurcation and chaos in power systems. The behaviour of a power system undergoes qualitative changes, i.e. bifurcations under system disturbances and parameter variations. These non-linear phenomena are undesirable and the studies by several researchers such as Chiang et al. [3], Srivastava et al. [4] and Rosehart et al. [6] illustrate that power system stability is governed by these non-linear phenomena. Quasi-periodic route to chaos has also been shown to exist in power systems [5]. Researchers have also attempted to control/avoid the occurrence of these phenomena. Wang et al. [7] employed a feed back law to control the amplitude and stability of periodic orbit. Srivastava et al. [8] have demonstrated that flexible AC transmission system (FACTS) devices such as controllable series capacitors (CSC), controllable phase angle regulator (PAR) and static VAR compensator (SVC) can be effectively used to damp out the Hopf bifurcation.

Bifurcations in underlying dynamical model of a power system are closely linked with its instability. This is especially true for voltage collapse precipitated by slow variation in a system parameter such as load. A power system model usually has multiple equilibria due to multiple solutions to load flow equations. Multiple equilibria in the governing dynamic model of a power system affect its stability. The unstable manifold of the controlling unstable equilibrium points on the stability boundary governs the region of attraction of stable equilibrium point. While stability is a single entity, the nature of instability arising in power systems has been broadly categorized as rotor angle instability or voltage instability. The unstable trajectory, following the instability, can illustrate the nature of the instability. The unstable manifold of the controlling unstable equilibrium point can approximate the unstable trajectory, after some finite time. Hence, the study of unstable manifold of the controlling unstable equilibrium points is important to determine the nature of instability.

The region of attraction of stable equilibrium point (s.e.p.) is decided by unstable equilibrium points (u.e.p.) and closed orbits (especially the type-1) lying on the stability boundary of the stable operating point. An unstable equilibrium point is said to be type-1 if it has only one eigenvalue lying in the right half plane and all

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other eigenvalues are lying in the left half plane. Once the instability sets in, the unstable manifold of the controlling unstable equilibrium point can approximate the unstable trajectory [9]. The study of unstable manifold of a power system due to a fault cleared immediately after the critical clearing is meaningful, for example in stability analysis and in relay setting. Reference [10] provides a good deal of theory and numerical computation techniques for determining one-dimensional unstable manifolds.

In this paper, the unstable manifold of unstable equilibrium points has been studied to demonstrate the difference between angle and voltage stability. The two sample power systems namely a 3-bus system and a 9-bus system, have been taken up for numerical investigations. In addition, the stability region in the parameter space has been characterised considering limit points (LP) saddle-node bifurcation (SNB) and Hopf bifurcation (HB).

2. Power system model

The stator transients are neglected and the transmission lines, modelled by their lumped π equivalent parameters in this study. The models of other dynamical components are as follows:

2.1. Generator model

The generators are modeled by a constant e.m.f. behind the transient reactance. The swing equations used for modeling the dynamics of an ith generator can be written as follows:

$$\frac{d\delta_i}{dt} = \omega_i \tag{1}$$

$$M_i\frac{d\omega_i}{dt} = P_{Mi} - P_{gei} - D_{gi}\omega_i \tag{2}$$

where δ is the load angle, ω the angular speed deviation from the synchronous speed, M the moment of inertia, P_M the mechanical input power, P_{ge} the output electrical power and D_g the damping coefficient.

2.2. Load model

The load characteristics have profound effect on the system dynamics and the classical models of load such as constant P and Q, constant current or constant impedance are not sufficient to capture the dynamics under severe disturbances [11]. Hence, in this study the model represents a composite load consisting of static and dynamic components. These load models are valid even for static analysis.

2.2.1. Load model — I

This load model considers a composite load consisting of a dynamic induction motor ($P_{IM} + jQ_{IM}$) in parallel with a static constant P, Q load ($P_s + jQ_s$). The model of induction motor has been taken from ref. [11] and expressed as a function of rate of change of its terminal voltage magnitude (V) and angle (θ). The model is given as follows:

$$P_s = P_1 \tag{3}$$

$$Q_s = Q_1 \tag{4}$$

$$P_{IM} = P_0 + K_{p\omega}\frac{d\theta}{dt} + K_{pv}\left(V + T\frac{dV}{dt}\right) \tag{5}$$

$$Q_{IM} = Q_0 + K_{q\omega}\frac{d\theta}{dt} + K_{qv}V + K_{qv2}V^2 \tag{6}$$

where P_0 and Q_0 are constant part of induction motor load, $K_{p\omega}$, K_{pv}, T, $K_{q\omega}$, K_{qv} and K_{qv2} are load coefficients.

2.2.2. Load model — II

If V and θ be the bus voltage magnitude and angle, respectively, the equations describing the load ($P_d + jQ_d$) are as follows [12]:

$$P_d = \ell\cos\phi + D_l\frac{d\theta}{dt} + a\frac{dV}{dt} \tag{7}$$

$$Q_d = \ell\sin\phi + b\frac{d\theta}{dt} + k\frac{dV}{dt} \tag{8}$$

where ℓ is the nominal MVA demand at the load bus, ϕ the power factor angle. D_l, a, b and k are the dynamic load parameters.

3. Case studies and discussion

Static bifurcations arise when number of steady state solutions changes if a system parameter is varied. In order to illustrate the occurrence of static bifurcations and its implications, studies were conducted on two sample systems namely a 3-bus and a 9-bus power system derived from ref. [13] and [14], respectively. The studies were assisted by AUTO86 software [15] on HP-9000/735 computer systems and are presented below.

3.1. 3-Bus system

Fig. 1 shows the topology of 3-bus system consisting of an infinite power source, a generator and a load bus. The generator is modelled by equations Eq. (1) and Eq. (2) and load by load model — I given by equations Eqs. (3)–(6).

This formulation results in a system of purely ordinary differential equations consisting of four state vari-