Applied Soft Computing 11 (2011) 2102–2116

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

On the continuous-time Takagi–Sugeno fuzzy systems stability analysis

Iman Zamani*, Mohammad Hadad Zarif

Department of Electrical Engineering, Shahrood University of Technology, Shahrood, Iran

ARTICLE INFO

Article history:
Received 11 January 2008
Received in revised form
14 September 2009
Accepted 25 July 2010
Available online 4 August 2010

Keywords:
Continuous-time T-S fuzzy systems
Stability analysis
Bilinear matrix inequality (BMI)
Parametric uncertainties

ABSTRACT

In this paper, a new approach for the stability analysis of continuous-time Takagi–Sugeno (T-S) fuzzy system is proposed. The universe set is divided to subregions, and piecewise quadratic Lyapunov function is then found for each of them. This class of Lyapunov function candidates is much richer than the common quadratic Lyapunov function. By exploiting the piecewise continuous Lyapunov function, we derive stability conditions that can be verified via convex optimization over linear matrix inequalities (LMIs) or bilinear matrix inequalities (BMIs). These conditions are shown to be less conservative than some quadratic stabilization conditions published recently in the literature. Since this method uses low numbers of LMIs or BMIs and less computation Lyapunov functions, it is highly applicable and has less computation. This approach is not dependent on the shape of fuzzy sets and also, stability of the system is guaranteed in the presence of state feedbacks. At first, in order to decrease length of computation (amount of LMIs (BMIs)), an approach is introduced based on properties of the T-S system with two-overlapped fuzzy sets. Some criterions are obtained for stability analysis, stability analysis in the presence of parametric uncertainties and a stability criterion is presented to provide a reasonable performance for the system. To demonstrate the new approach, an illustrative example is presented.

1. Introduction

Fuzzy sets and systems were developed and extensively applied in previous decades [30] and relevant methods were developed and applied with more or less success depending on the specific problem, and recently, fuzzy control has been successfully applied to a variety of industrial processes. Certainly, without carrying out an in-depth analysis, the design of the system and controller may come with no guarantee of system stability. So, performance and stability are important aspects of designing these kinds of systems. Based on these remarks, researches, application of fuzzy systems and this fact that any nonlinear systems can be modeled by a fuzzy system [6], fuzzy model can be a reasonable model of nonlinear systems in huge number of modeling problems.

Among these models, Takagi–Sugeno–Kang (T-S) system is mentioned which was presented by [21] as a new fuzzy system after advent of Mamdani model. Because of linear state form of consequents in the rules of this system and applicable of many linear control theory, this model (i.e. T-S) was prospered at a high rate. Thereafter, Parallel Distributed Compensation (PDC) technique was presented in [22] as a new method for designing and controlling for such type of fuzzy systems. Arrival of these type of fuzzy models required new methods for stability analysis and controlling subsequently. Extended and innovative methods and relevant conditions for stability analysis of continuous and discrete fuzzy systems were found. Some methods and conditions for stability in continuous and discrete fuzzy systems are presented in [4,10,19,20,23,28]. There is also shown some controlling methods based on PDC in [15,24,25]. Among other researches, we can mention a type of nonlinear controllers for fuzzy systems was presented by [8,31] which have low computation, a new approach based on concept of fuzzy positive definite and negative definite function that proposed by [17,18] and other approaches based on Lyapunov function and LMIs (BMIs) as important role in the synthesis of T-S fuzzy systems [26,29].

In this evolution, most of results require the existence of a common quadratic Lyapunov function. For instance Tanaka and Sugeno proposed a design and stability method for fuzzy systems via Lyapunov direct method [22]. A common positive definite symmetric matrix P must be found to satisfy the Lyapunov equation for all local linear models. In many cases, it is difficult to find a matrix P when the number of rules for fuzzy system is large and conditions for existence of such functions are restrictive and difficult to establish. Also, some of these methods cannot be used directly for designing because of pre-designed feedback gains (e.g. [15]) and the method must be to check stability for pre-designed system that requires trial-and-error for control design. Another important observation is that the length of computation

* Corresponding author. Tel.: +98 21 64543398.
E-mail addresses: zamaniiman@aut.ac.ir, zamaniiman@gmail.com (I. Zamani), mhzarif@shahroodut.ac.ir (M.H. Zarif).

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doi:10.1016/j.asoc.2010.07.009

and the number of LMIs (BMIs) should be low. Some researches and literatures present ideas and approaches which have high computation that make these methods unusable just as they are innovative and fair [4,22]. Another important subject that should be attended is stability analysis with desired performance. Surely, much attempts focus to remove these faults and flaws [18,20], however, it is still necessary to find a practical method to guarantee stability and provide performance. So, it is expected to continue growing in this field at an estimated rate of high in coming years.

The contributions of this paper are organized as follows: After introduction of T-S systems, in Section 3, we introduce the concept of Operating Subregion (*OS*) and Two-Overlapped fuzzy sets, then an approach is proposed for decreasing the number of LMIs (BMIs). For showing the benefits of this approach, we shall give a comparison with known methods. In the second part of this section, this approach is used to obtain sufficient conditions for stability analysis, stability analysis in presence of parametric uncertainties and so on, stability analysis with reasonable performance together.

In Section 4, an illustrative example will be given to verify the result of this paper. Finally, results are argued in Section 5.

Nomenclature. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, the n dimensional Euclidean space and the set of all $n\times m$ real matrices respectively. The superscript "T" denotes "matrix transpose" and the notation $X\geq Y$ (respectively, $X>Y$) where X and Y are matrix, means that $X-Y$ is positive semi-definite (respectively, positive definite). $\dot{V}$ denotes $\partial V/\partial t$ where V is a $n\times 1$ function vector with respect to argument t.

$||A||_p$ means *p-norm* of matrix A. "*min*" and "*max*" are abbreviations for minimum and maximum respectively, $(\lambda_{min}(A), \lambda_{max}(A))$ denotes to (minimum Eigen value of A, maximum Eigen value of A), consequently. Also, $|a|$ means absolute value of a and ":=" indicates "defined as".

2. Preliminaries

In this section, we introduce the concepts and definitions we need to introduce T-S fuzzy systems. The basic idea of fuzzy modeling for T-S fuzzy models is to decompose the input space into a number of fuzzy regions in which the systems behavior is approximated by local linear model. The overall fuzzy model is then a fuzzy blending of the local model interconnected by a set of membership functions. In the continuous case, T-S fuzzy model can be described by the following fuzzy rules:

Plant rules:

$$\begin{aligned}&\textit{if } p_1(t) \textit{ is } M_1^i \textit{ and } p_2(t) \textit{ is } M_2^i \textit{ and} \ldots \textit{and } p_s(t) \textit{ is } M_s^i\\ &\textit{then} \begin{cases}\dot{x}(t)=A_ix(t)+B_iu(t)\\ y_i(t)=C_ix(t)\end{cases} \quad (i=1,2,\ldots,r)\end{aligned} \tag{1}$$

where r is the number of fuzzy rules, and $p(t)$ is vector of premise variables such that $p(t)=[p_1(t), p_2(t), \ldots, p_s(t)]^T=\mathcal{O}x(t)$, $rank(\mathcal{O})=s(1\leq s\leq n)$. $x(t)=[x_1(t), x_2(t), \ldots, x_n(t)]^T\in\mathbb{R}^n$ is state vector at time t, n is the number of states variable, (A_i, B_i) are the system matrices that are controllable, $y_i(t)$ is output of ith subsystem and C_i is output gain.

$u(t)=[u_1(t), u_2(t), \ldots, u_m(t)]^T\in\mathbb{R}^m$ is control input at time t with appropriate dimension, M_j^i $(i=1,2,\ldots,r; j=1,2,\ldots,s)$ stands for the fuzzy set of jth antecedent variable in the ith rule.

By the singleton fuzzifier, product inference and the center average defuzzifier, the final output of the fuzzy systems can be represented as:

$$\dot{x}(t)=\sum_{i=1}^{r}\left(\left[\frac{\omega_i(p(t))}{\sum_{j=1}^{r}\omega_j(p(t))}\right](A_ix(t)+B_iu(t))\right) \tag{2}$$

where $\omega_i(p(t))=\prod_{j=1}^{s}(\mu_{M_j^i}(p(t)))$ and $\sum_{j=1}^{r}\omega_j(p(t))\neq 0$ for all $t\geq 0$. $\mu_{M_j^i}(p(t))$ is grade of $p(t)$ by M_j^i. Based on the PDC technique [22], the following control laws are usually employed for the stabilization of T-S fuzzy models:

Controller rules:

$$\begin{aligned}&\textit{if } p_1(t) \textit{ is } M_1^i \textit{ and } p_2(t) \textit{ is } M_2^i \textit{ and} \ldots \textit{and } p_s(t) \textit{ is } M_s^i\\ &\textit{then} \quad u(t)=K_iy_i(t) \quad (i=1,2,\ldots,r)\end{aligned} \tag{3}$$

where $K_i(i=1,2,\ldots,r)\in\mathbb{R}^{m\times n}$ are the output feedback gains to be designed. Consequently, the overall fuzzy output feedback controller law can be expressed as:

$$u(t)=\sum_{i=1}^{r}\left[\frac{\omega_i(x(t))}{\sum_{j=1}^{r}\omega_j(x(t))}\right]K_iy_i(t) \tag{4}$$

For the sake of convenience we denote

$$\left(\frac{\omega_i(p(t))}{\sum_{j=1}^{r}\omega_j(p(t))}\right):=\alpha_i(p(t)) \tag{5}$$

Obviously it holds: $0\leq\alpha_i(p(t))\leq 1$ for all $i=1,2,\ldots,r$ and $\sum_{i=1}^{r}\alpha_i(p(t))=1$. In general, $\alpha_i(p(t))$ can be regarded as the matching degree between the state variable and the antecedent of the ith fuzzy rule. By substituting $u(t)$, we obtain the following formulation of the closed-loop models:

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(p(t))\alpha_j(p(t))(A_i+B_iK_jC_j)x(t) \tag{6}$$