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Frequency parameterization of H_∞ PID controllers via relay feedback: A graphical approach

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ABSTRACT

This paper focuses on a graphical approach to determine the region of proportional-integral-derivative (PID) controllers in the parameter space for which the closed-loop system is internally stable and the H_∞ optimization criteria are satisfied for a class of single-input single-output arbitrary order plant with or without dead time. Unlike conventional methods which the analytical models, such as transfer functions and state space models, are needed, the design information of the proposed approach is only the frequency response data, which are directly calculated from a single relay test for stable plants, or extracted from the closed-loop system frequency response data by dividing out the known stabilizing compensator for unstable plants using relay feedback methods. It is shown that the problem to be solved can be translated into simultaneous stabilization of the closed-loop characteristic function and a family of characteristic functions. Based on the technique of *D*-decomposition, the analytical boundaries of root invariant regions are derived and the admissible H_∞ region in the parameter space is the intersection of the admissible sets, and it can be drawn and identified immediately, not to be computed mathematically. A practical algorithm of determining the H_∞ region is proposed and two examples are used to illustrate the proposed method.

1. Introduction

In recent years, an interesting research in the direction of finding ways of computing all stabilizing regions in the parameter space of PID controllers has captured the attention of several researchers [1–10]. This fact is that the PID controller has just three tuning parameters, and the parameter space approach is suitable for the control design. Over the last 40 years control theory literature has been dominated by modern optimal control theory and its offshoots, such as the powerful H_∞ theory [11–13], but the resulting controllers may be of a very high order, sometimes even exceeding that of the original system. Since the H_∞ theory does not allow one to constrain the order of the designed controller, its direct application to the design of controllers of a given structure, such as PID controllers, faces substantial difficulties. Fortunately, there have been many researchers in attempting to combine the power of optimal control with low order/fixed structure controllers [14–18]. For example, based on the generalization of the Hermite–Biehler theorem, Ho [15] proposed a linear programming characterization of all admissible H_∞ PID controllers for a given plant. Keel and Bhattacharyya [19] developed a technique to compute the set of all first order stabilizing controllers which satisfy an H_∞ constraint for a given but arbitrary linear time-invariant plant based on determination of root invariant regions via *D*-decomposition and parameter mapping. An alternative approach to the design of low order controllers of a given structure which satisfy the H_∞-criterion was presented in [18]. But these methods rely on the fact that plant parameters are known, i.e., analytical models of plants, such as transfer function or state space models, must be constructed first.

However, in real world systems, there exist many situations where such exact information of modeling is unavailable or is difficult to obtain in practical applications. On the other hand, it is often the case that the time series data or frequency response of the plant can be easily measured experimentally. Recently, the approaches in [10,19,20] were developed to determine the entire set of stabilizing first-order/PID controller only based on the frequency response of a given LTI plant. Based on the concept of *D*-decomposition, the methods of determining the stabilizing region of PID-type controllers and first order controllers were proposed in [10] and [19], respectively. Using the sign change properties and the generalized Hermite–Biehler theorem under some reasonable assumptions, a method of computation of all stabilizing PID controllers was given in [20]. But the above results cannot be directly applied to the plants with dead time. Lately, in [21], it is shown that the entire sets of three term controllers achieving stability and various performance specifications can be found from the Nyquist–Bode data

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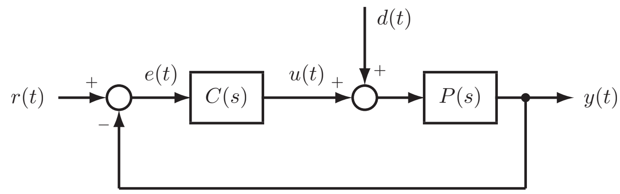

Fig. 1. PID feedback control system.

using linear programming with a sweeping parameter under some standing assumptions. However, for the approaches based on the generalized Hermite–Biehler theorem by fixing the K_p gain and determining admissible regions in $K_i - K_d$ plane, there is need to solve a great amount of equations for finding singular frequencies to obtain a set of linear inequalities.

This note is motivated for the consideration of the case of the analytical models, such as transfer functions or state space models, which are unavailable or are difficult to obtain, but the plant frequency response data determined from a single relay test at hand, and the case of the generalization of frequency parameterization of PID controllers satisfying multiple H_∞ optimization criteria for a class of arbitrary order plants with or without time delay.

The important properties of the proposed method are: (1) it does not rely on any analytical models, and the only information for the designers is the frequency response data, which are directly calculated from a single relay test for stable plants, or extracted from the closed-loop system frequency response data by dividing out the known stabilizing compensator for unstable plants using relay feedback methods; (2) it is shown that the exact boundaries can also be identified if only the plant frequency response is measured accurately in a finite frequency band since the analytical boundaries of characterization of all H_∞ PID controllers are derived; (3) it is efficient and fast, and there is no need to solve a great amount of equations for finding singular frequencies to obtain a set of linear inequalities, then using the generalized Hermite–Biehler theorem to check the strings formed by singular frequencies. The finite number of domains in the parameter space of PID controllers needed to be checked is reduced to an ad hoc low level and it will be seen in the sequel that, in most cases, there is only one region need to be tested and thus as such can be used in near real-time to design PID controllers for a wide range of real world systems.

The paper is organized as follows: In Section 2 some statements including the PID control system, the design information and the H_∞ optimization criteria are introduced. The approach of estimating multiple points on the process frequency response and the method to determine the characteristic parameters only from the frequency response data for design of PID controllers are addressed in Section 3. Then we briefly review the basic idea of D-decomposition in Section 4. Section 5 is devoted to the computation of the entire set of stabilizing PID controllers in the parameter space. The guidelines for choosing the desired H_∞ levels of γ_s, γ_t and γ_d are proposed in Section 6. The synthesizing H_∞ PID controllers are considered and a practical algorithm of determining the H_∞ region is presented in Section 7. Then, in Section 8, a unified design framework of determining the desired H_∞ regions of PID controllers is given. Finally, two examples are illustrated in Section 9 and Section 10 contains some conclusions and discussions.

2. Problem formulation

2.1. PID control system

Consider the unit feedback of PID control system shown in Fig. 1, where $r(t)$ is the reference input, $y(t)$ is the system output, $d(t)$ is the disturbance, $u(t)$ is the control signal and $e(t)$ is the control error. This system consists of a SISO, LTI, strictly proper, plant $P(s)$, and a PID controller $C(s)$ given by

$$P(s) = \frac{N(s)}{D(s)} e^{-\tau s} \quad \text{and} \quad C(s) = K_p + \frac{K_i}{s} + K_d s \tag{1}$$

where τ is the dead time, $\tau \geq 0$ and K_p, K_i and K_d are the proportional, integral and derivative gains, respectively. $N(s)$ and $D(s)$ are coprime polynomials and given by

$$\begin{cases} N(s) = a_m s^m + \cdots + a_1 s + a_0 \\ D(s) = s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0 \end{cases}$$

In particular, the following three closed-loop transfer functions are considered:

- The sensitivity function:

$$S(s) = \frac{1}{1 + C(s)P(s)} \tag{2}$$

- The complementary sensitivity function:

$$T(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} \tag{3}$$

- The disturbance sensitivity function:

$$G_d(s) = \frac{P(s)}{1 + C(s)P(s)} \tag{4}$$

2.2. Design information

The frequency response of $P(s)$ can be described in the frequency domain by the relation

$$P(j\omega) = \frac{N(j\omega)}{D(j\omega)} e^{-j\omega\tau} = P_a(\omega) e^{j\varphi(\omega)} = P_r(\omega) + jP_i(\omega) \tag{5}$$

where $P_r(\omega)$ and $P_i(\omega)$ are the real and imaginary parts of the frequency response of the plant, respectively. $P_a(\omega)$ is the magnitude and $\varphi(\omega)$ is the phase of the plant, and

$$P_a^2(\omega) = P_r^2(\omega) + P_i^2(\omega) \tag{6}$$

In practice, the estimation of multiple points on the process frequency response of $P(j\omega)$ can be obtained from a relay test [22–25].

First, we define η as a time delay judgement factor. If the plant has a time delay $\tau > 0$ then $\eta = 1$; otherwise, $\eta = 0$, the plant is a delay-free plant and $\tau = 0$. Then the following assumptions throughout the paper are listed as below:

Assumption 1. The information for the designers:

(1) The plant has no poles on the imaginary axis except for possible several ones at the origin.
(2) Knowledge of the plant whether which contains a time delay or not, i.e., knowledge of the factor η.
(3) Knowledge of a known stabilizing controller if the plant is unstable.

Remark 2. For the plant containing a time delay, the exact value of τ is not needed to determine in the proposed method, and which accords with the fact in real world systems that it is very difficult or impossible to determine the exact value of τ only from experiment data due to disturbances and noises. Comparatively, it is easy to determine the factor of η. In practice, the value of η may be known from prior physical considerations or the value of η may be ascertained from the experimentally determined step response.

Remark 3. For a stable LTI plant, the number of RHP poles, denoted as κ^+, and $\kappa^+ = 0$, and the frequency response data of the plant can be estimated directly from a single relay test. For an unstable plant,