



## Research Article

## On the fragility of fractional-order PID controllers for FOPDT processes

Fabrizio Padula<sup>a</sup>, Antonio Visioli<sup>b,\*</sup><sup>a</sup> Dipartimento di Ingegneria dell'Informazione, University of Brescia - Italy, Italy<sup>b</sup> Dipartimento di Ingegneria Meccanica e Industriale, University of Brescia - Italy, Via Branze 38, I-25123 Brescia, Italy

## ARTICLE INFO

## Article history:

Received 2 December 2014

Received in revised form

26 August 2015

Accepted 9 November 2015

Available online 27 November 2015

This paper was recommended for publication by Dr. Y. Chen

## Keywords:

Fractional-order controllers

PID control

Tuning

Fragility

## ABSTRACT

This paper analyzes the fragility issue of fractional-order proportional-integral-derivative controllers applied to integer first-order plus-dead-time processes. In particular, the effects of the variations of the controller parameters on the achieved control system robustness and performance are investigated. Results show that this kind of controllers is more fragile with respect to the standard proportional-integral-derivative controllers and therefore a significant attention should be paid by the user in their tuning.

© 2015 ISA. Published by Elsevier Ltd. All rights reserved.

## 1. Introduction

It is well known that a properly designed control system must provide an effective trade-off between performance and robustness. However, it has also been recognized that another important issue to be addressed is the fragility of the control system to the variation of the controller parameters, that is, the sensitivity of the robustness and/or performance of the control system to changes in the controller parameters.

This issue has been raised in the literature in some papers (see, for example, [1]) and, in particular, in [2] where it has been stressed that design techniques based on the minimization of the  $H_2$ ,  $H_\infty$  and  $l_1$  norms can yield to high-order robust, optimal but also extremely fragile controllers, namely, a very small variation of the controller coefficients can result in an unstable system. However, in [3,4] it has been pointed out that this problem can be solved by using a suitable controller parametrization.

As integer-order proportional-integral-derivative (IOPID) controllers are the most used controllers in industry, the fragility of such a kind for controllers has been specifically addressed in [5,6]. Therein, authors suggest to tune the IOPID controller in order to maximize the  $l_2$  norm of the controller parameter vector in the stabilizing region for a given plant. However, the typical industrial performance measures (related to the set-point

following and/or to the load disturbance rejection task) are not taken into account. Further, it has been shown in [7] that this kind of approach applied to first-order-plus-dead-time (FOPDT) and integrator-plus-dead-time (IPDT) processes yields a tuning similar to that obtained by using the Ziegler–Nichols step response method [8] which is known to be improvable under many points of view [9].

Thus, it has been recognized in the literature that one of the main reasons to investigate the fragility of IOPID controllers is to give to the user an idea of how a fine tuning of the controller can be done [10–12]. In other words, as the IOPID parameters have a clear physical meaning, the operator can modify them in order to change the control system performance. In this context, it is useful to evaluate the sensitivity of the robustness/performance behavior with respect to (small) changes of the parameters. For this purpose, a graphic tool called fragility rings providing a visual aid for evaluation of the controller robustness/fragility has been proposed in [13].

In the recent years, there has also been a significant interest from the academic and industrial communities for fractional-order-proportional-integral-derivative (FOPID) controllers because they are capable to provide (as there are five parameters to tune) more flexibility in the control system design (see, for example, [14–17]). Many different tuning rules have been proposed in the literature to facilitate their use (see, for example, [18–23]). In this context, while the problem of stabilizing a (possibly fractional) dynamic system using FOPID controllers has been already addressed in the literature (see, for example, [24–26]), for such a

\* Corresponding author. Tel.: +39 030 3715460; fax: +39 030 380014.

E-mail addresses: [fabrizio.padula@unibs.it](mailto:fabrizio.padula@unibs.it) (F. Padula), [antonio.visioli@unibs.it](mailto:antonio.visioli@unibs.it) (A. Visioli).

kind of controllers, a fragility analysis has been only partially exploited until now. In particular, in [27,28], the tuning of the FOPID controllers is performed by considering the centroids of the admissible regions in the parameter space so that a non-fragile controller results. However, one of the main purposes for evaluating the fragility of the controller is in evaluating the sensitivity of the robustness/performance indexes to the (possibly fine) tuning of the parameters.

Indeed, in order to foster a widespread use of FOPID controllers in industrial plants, in addition to well-established tuning rules, clear guidelines on how to modify the controller parameters should be given to the operator in order for him/her to be confident with them. Thus, the aim of this paper is to provide a fragility analysis for FOPID controllers and to make a comparison with IOPID controllers in order to understand the differences that should be taken into account in the adjustment of the parameters starting from a given tuning. For this purpose, the tuning rules proposed in [23,29], which aim at minimizing the integrated absolute error subject to constraints on the maximum sensitivity, are used, both for FOPID and IOPID controllers. Both the tuning rules for the set-point following and the load disturbance rejection tasks are considered. They also have the significant feature of providing a control action that is invariant when the time unit is changed. These tuning rules are therefore suitable to perform a fragility analysis with respect to both robustness and performance. It is worth stressing that the calculated fragility depends on the nominal parameters of the control system and for this reason, in order to obtain a fair comparison, we select tuning rules that solve the same optimization problem, so that the possible additional complexity of adjusting the parameters of a FOPID controller, with respect to a IOPID one, starting from a given tuning is clearly addressed.

The fragility is evaluated by changing all the parameters at the same time or just one of them by keeping the other ones fixed. The latter case is performed in order to investigate which parameter has more influence on the controller fragility.

The paper is organized as follows. The basic definitions employed for the fragility evaluation are reviewed in Section 2, in

addition to the description of the tuning rules used for both integer-order and fractional-order PID controllers. The fragility analysis related to the robustness is presented in Section 3 while that related to the performance is presented in Section 4. A discussion is made in Section 5, while conclusions are drawn in Section 6.

## 2. Fragility indices

The fragility indices proposed in [10–12] are briefly reviewed in this section for the sake of clarity and in order to introduce the notation used in presenting the results.

Consider a unity feedback control system (see Fig. 1) where the process (which is assumed to be self-regulating) is denoted as  $P$  and the controller as  $C$ . In this paper, the controller is a FOPID controller, which can be expressed either in series form, *i.e.*,

$$C(s) = K_p \frac{T_i s^\lambda + 1}{T_i s^\lambda} \frac{T_d s^\mu + 1}{T_f s + 1} \tag{1}$$

or in parallel (ideal) form, *i.e.*,

$$C(s) = K_p \left( 1 + \frac{1}{T_i s^\lambda} + T_d s^\mu \right) \frac{1}{T_f s + 1}. \tag{2}$$

In both expression,  $K_p$  is the proportional gain,  $T_i$  is the integral time constant,  $T_d$  is the derivative time constant and  $\lambda$  and  $\mu$  are the noninteger orders of the integral and derivative terms respectively.

Note that it is important to consider both forms (1) and (2) because it is not possible to transform (2) into an equivalent form (1) and *vice versa* unless  $T_i \geq 4T_d$  and  $\lambda = \mu$  [29]. In order to implement the fractional-order controller, the well-known Oustaloup continuous integer-order approximation [30] has been employed to approximate the fractional differintegrator. In this paper 16 poles and zeros have been used in order to approximate the fractional differintegrator in a frequency range  $[\omega_l, \omega_h]$ , where  $\omega_l$  and  $\omega_h$  have been selected as  $0.0001\omega_c$  and  $10000\omega_c$  respectively, with  $\omega_c$  being the gain crossover frequency. It is worth noting that the used number of poles and zeros leads to a computationally demanding controller and, actually, the fractional controller could be approximated with a lower order integer one. Nevertheless, considering that the purpose of this paper is the fragility analysis of the fractional controller, a higher computational cost is accepted in order to achieve an improved approximation. The approximated and the ideal open loop transfer function in this way are virtually indistinguishable at those frequencies that have an appreciable impact on the closed-

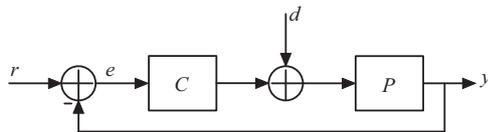


Fig. 1. The considered control scheme.

Table 1

The controller parameters for the considered example with  $L/T = 0.5$  and for the different control tasks, set-point (SP) following and load disturbance (LD) rejection with a maximum sensitivity of 1.4 and 2.0 respectively.

Controller	$K_p$	$T_i$	$T_d$	$\lambda$	$\mu$
FOPID series					
SP 1.4	1.1060	0.9839	0.1554	1	1.2
SP 2.0	1.6698	1.0281	0.1975	1	1.1
LD 1.4	0.7818	0.4683	0.2617	1	1.1
LD 2.0	1.1182	0.4236	0.3105	1	1.1
FOPID parallel					
SP 1.4	1.3307	1.1765	0.1384	1	1.1578
SP 2.0	2.0850	1.2507	0.1757	1	1.1351
LD 1.4	1.2786	0.8824	0.1686	1	1.1351
LD 2.0	2.3611	0.9079	0.1440	1	1.1525
IOPID series					
SP 1.4	0.8676	0.8127	0.2074	–	–
SP 2.0	1.4708	0.9568	0.2347	–	–
LD 1.4	0.6369	1.0081	0.3031	–	–
LD 2.0	1.007	0.4106	0.3304	–	–

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات