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Comments on "Stability analysis for a class of Takagi–Sugeno fuzzy control systems with PID controllers"

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ABSTRACT

This note points out that some results in the paper (International Journal of Approximate Reasoning 46 (2007) 109–119) are incorrect.

1. Introduction

In the paper [2], a new fuzzy T–S transfer function model is proposed, and the relation between the classical T–S fuzzy model and the proposed model is discussed in Lemma 1, which is cited as follows:

Lemma 1. *The T–S fuzzy state model (1) (numbered in [2]) with the following Frobenius canonical structure*

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0^i & -a_1^i & \cdots & \cdots & -a_{n-1}^i \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0^i \end{bmatrix}, \quad C = [1 \quad 0 \quad \cdots \quad \cdots \quad 0], \tag{1}$$

where $A_i \in R^{n\times n}, B_i \in R^{n\times 1}, C \in R^{1\times n}$ is equivalent to the T–S fuzzy transfer function model (11) (numbered in [2]) with $b_j^i = 0$, $i = 1,\ldots,r, j = 1,\ldots,m$.

The original proof is rewritten as follows.

Proof. The overall aggregated model of (11) (numbered in [2]) is

$$P(s) = \frac{\sum_{i=1}^{r} \alpha_i b^i(s)}{\sum_{i=1}^{r} \alpha_i a^i(s)} = \frac{Y(s)}{U(s)}. \tag{2}$$

Denoting the following state variables

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$$x_1 = y, x_2 = \dot{y}, \ldots, x_n = y^{(n-1)}, \tag{3}$$

we have

$$\dot{x}_n = -\sum_{i=1}^{r}\alpha_i(x)a_0^i x_1 - \sum_{i=1}^{r}\alpha_i(x)a_1^i x_2 - \cdots - \sum_{i=1}^{r}\alpha_i(x)a_{n-1}^i x_n + \sum_{i=1}^{r}\alpha_i(x)b_0^i u. \tag{4}$$

This is the matrix form (5) (numbered in [2]) with the above mentioned matrices A_i, B_i, C in (16) (numbered in [2]). □

2. Discussion

In our opinion, the Lemma 1 is incorrect. As a matter of fact, if $\alpha_i(\cdot)$ is independent of X, where $X = [x_1, x_2, \ldots, x_n]^T$, with all initial conditions being zero, the Laplace Transform [1] of Eq. (4) is of the following form:

$$s^n Y(s) = -\sum_{i=1}^{r}\alpha_i a_0^i Y(s) - \sum_{i=1}^{r}\alpha_i a_1^i sY(s) - \cdots - \sum_{i=1}^{r}\alpha_i a_{n-1}^i s^{n-1}Y(s) + \sum_{i=1}^{r}\alpha_i b_0^i U(s). \tag{5}$$

With the following assumption

$$\sum_{i=1}^{r}\alpha_i = 1, \tag{6}$$

Eq. (5) can be recast as

$$\sum_{i=1}^{r}\alpha_i s^n Y(s) = -\sum_{i=1}^{r}\alpha_i a_0^i Y(s) - \sum_{i=1}^{r}\alpha_i a_1^i sY(s) - \cdots - \sum_{i=1}^{r}\alpha_i a_{n-1}^i s^{n-1}Y(s) + \sum_{i=1}^{r}\alpha_i b_0^i U(s), \tag{7}$$

$$\sum_{i=1}^{r}\alpha_i(a_0^i + a_1^i s + \cdots + a_{n-1}^i s^{n-1} + s^n)Y(s) = \sum_{i=1}^{r}\alpha_i b_0^i U(s), \tag{8}$$

$$\sum_{i=1}^{r}\alpha_i a^i(s)Y(s) = \sum_{i=1}^{r}\alpha_i b^i(s)U(s), \tag{9}$$

where $a^i(s) = s^n + a_{n-1}^i s^{n-1} + \cdots + a_0^i$, and $b^i(s) = b_m^i s^m + \cdots + b_1^i s + b_0^i$ with $b_j^i = 0$, $i = 1, \ldots, r$, $j = 1, \ldots, m$ which is defined in (11) of the paper [2]. Moreover, we obtain

$$\frac{Y(s)}{U(s)} = \frac{\sum_{i=1}^{r}\alpha_i b^i(s)}{\sum_{i=1}^{r}\alpha_i a^i(s)}. \tag{10}$$

However, as for the T–S fuzzy model, $\alpha_i(\cdot)$ is in no way independent of X, which means (4) is a nonlinear differential equation. In this case, we can not obtain (10) from (4) for the simple reason that in general the following equation does not hold:

$$\mathscr{L}[f_1(t)f_2(t)] = f_1(s)f_2(s), \tag{11}$$

where the symbol '$\mathscr{L}$' represents the Laplace Transform; $f_1\ (t)$ and $f_2\ (t)$ are two arbitrary functions which are Laplace transformable.

This means that the T–S fuzzy state model (1) (numbered in [2]) with the Frobenius canonical structure as mentioned in Lemma 1 is *not* equivalent to the T–S fuzzy transfer function model at all. Thus we conclude that the Lemma 1 is incorrect.

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