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Iterative Procedure for Tuning Decentralized PID Controllers

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Abstract: An iterative procedure is proposed to design decentralized PID controllers for multi-loop processes. Each SISO loop is designed at a time, the controller parameters are computed by a convex optimization problem with constraints on stability margins. Despite the SISO approach, loops interactions are taken into account by Gershgorin bands and Equivalent Open-loop Process (EOP). Two simulation examples are presented to compare the performance with related techniques.

Keywords: PID, MIMO systems, Optimization, Gain and phase margins, Process control.

1. INTRODUCTION

The basic regulatory control layer of process plants largely consists of decentralized SISO PID controllers. The main reasons are: relatively simple structure, easy to maintain, fewer parameters to tune than full multivariable controllers, and loop failure tolerance. Despite the SISO approach, tuning of each loop cannot be done independently due to loops interactions. Applying the tuning methods for SISO systems ignoring interactions often leads to poor performance and stability.

One way to consider loop interactions during controller design is to use the Gershgorin bands (see e.g. Ho and Xu (1998), Chen and Seborg (2001), and Garcia et al. (2005)). Closed-loop stability can be ensured by shaping the Gershgorin bands in such a way that they do not overlap the critical point $(-1, 0)$ and encircle it the appropriate number of times, in accordance with the generalized Nyquist theorem. An advantage of this method is that knowledge of controller parameters from other loops is unnecessary. However, effectiveness of this method is restricted to diagonally dominant multi-loop processes.

Another way to take loop interactions into account to controller design is to use the concept of equivalent open-loop process (EOPs) (see e.g. Huang et al. (2003), Vu and Lee (2010), and Nie et al. (2011)). This procedure requires initial controller parameters for each loop. After all loops have been closed, the controller will then be retuned one after the other with all other loops being closed with the controllers obtained in the previous step. This procedure will go on until controllers parameters converge. Most methods need model order reduction when compute the EOP.

In this paper, an iterative procedure to tune decentralized PID controllers is proposed by taking other loops interactions into account both by Gershgorin bands as EOPs. The procedure consists of two main steps. In the first step, initial controllers are designed based on Gershgorin bands. The second step, which design is now based on EOPs, each controller is re-tuned iteratively until controller parameters converge. The SISO controller design method used in all steps is formulated as a convex optimization problem which minimizes the diagonal loop integral error subject to minimum stability margins.

With this proposed procedure some advantages are achieved compared to related methods. First, full Gershgorin bands are considered, different from Ho and Xu (1998), and Chen and Seborg (2001), that are based on no more than two Gershgorin circles, which does not guarantee overall system stability. Second, this procedure is not restricted to diagonally dominant processes, model reduction is not necessary, and high dimensional processes are not an issue. Third, lower bound on stability margins can be specified to each diagonal loop transfer function, independently how interactions are considered in design method.

2. PROBLEM STATEMENT

Consider the closed loop system of Figure 1, where $G(s) = [g_{ij}(s)]_{n\times n}$ is a $n\times n$ process, $C(s) = diag\{c_1(s), \ldots, c_n(s)\}$ is a decentralized controller, and $L(s) = [g_{ij}(s)c_j(s)]_{n\times n}$ is the multivariable loop-gain. The $\mathbf{r}$, $\mathbf{u}$, and $\mathbf{y}$ are the set-point, manipulated, and controlled variable vectors.

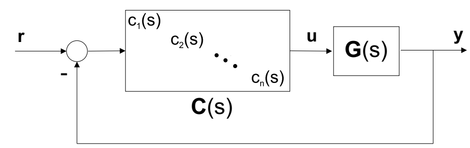

Fig. 1. Decentralized control system.

The PID controller transfer function from the j-th diagonal element of $C(s)$ is

$$c_j(s) = k_p + \frac{k_i}{s} + \frac{k_d s}{T_f s + 1} \quad (1)$$

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where k_p, k_i and k_d are the controller parameters and T_f is the time constant of noise filter, which is assumed known. This PID formulation could be written as:

$$c_j(s) = \rho^T \phi(s) \tag{2}$$

where $\rho^T = [\, k_p \; k_i \; k_d \,]$ and $\phi^T(s) = [\, 1 \; \frac{1}{s} \; \frac{s}{T_f s+1} \,]$. Furthermore, the dynamic of each element of $G(s)$ can be captured by a finite number of N frequency points, $g_{ij}(j\omega_k) \quad k = 1, \ldots, N$. With this parameterization, every point on the Nyquist diagram of $l_{jj}(j\omega_k)$ could be written as a linear function of the controller parameters:

$$l_{jj}(j\omega_k) = \rho^T \Re_{jj}(\omega_k) + j\rho^T \Im_{jj}(\omega_k), \tag{3}$$

where $\Re_{jj}(\omega_k)$ and $\Im_{jj}(\omega_k)$ are, respectively, the real and the imaginary parts of $\phi(j\omega_k)g_{jj}(j\omega_k)$.

3. THE MULTI-LOOP INTERACTIONS

Despite each $c_j(s)$ controller is designed by a SISO tuning method, the multi-loop interactions are taken into account to guarantee closed-loop stability. The proposed procedure is based on two approaches to consider loops interactions: Gershgorin bands and Equivalent Open-loop Process (EOPs).

3.1 The Gershgorin bands

Consider the Nyquist plot of $l_{jj}(j\omega) = g_{jj}(j\omega)c_j(j\omega)$ with a circle of the radius

$$R_j(\omega) = \sum_{i=1, i\neq j}^{n} |g_{ij}(j\omega)c_j(j\omega)|\,, \tag{4}$$

the Gershgorin circle centered on l_{jj}. When these Gershgorin circles are superimposed on the diagonal elements of the Nyquist array, they form the Gershgorin bands. In the Direct Nyquist Array method (Rosenbrock (1970)), controllers are designed by shaping the Gershgorin bands using a trial and error graphical approach. The following theorem states the stability of the closed loop.

Theorem 1. (Rosenbrock (1970)) Suppose the Gershgorin bands centered on the diagonal elements $l_{jj}(j\omega)$ of $L(j\omega)$, and $j = 1, \ldots, n$ exclude the point $(-1, 0)$. Let the j-th Gershgorin band encircles the point $(-1, 0)$ N_j times anticlockwise. Then, the closed-loop system is stable if, and only if,

$$\sum_{j=1}^{n} N_j = p_0 \tag{5}$$

where p_0 is the number of unstable poles of $L(j\omega)$.

Since most industrial process are open-loop stable, the controller design procedure assumes that $p_0 = 0$. Thus, Gershgorin bands must not encircle nor include the critical point $(-1, 0)$ to ensure stability.

3.2 The Equivalent Open-loop Process

Consider an equivalent open-loop process (EOP) as the transfer function that describes the effective dynamics of each open-loop from u_i to y_i while all other loops

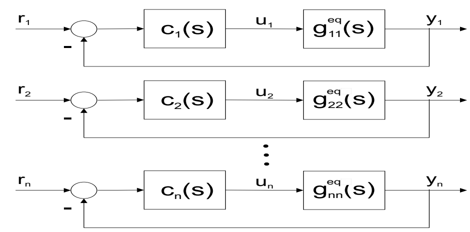

Fig. 2. SISO Systems with the corresponding EOP.

are closed. The EOP differs from the original open-loop transfer function by the interactions effects of the coupled loops, thus the EOP corresponds to the actual open-loop transfer function under multi-loop cases. Applying the EOP concept, the block diagram in Figure 1 can be decomposed into a set of n equivalent SISO systems in Figure 2.

The equation that defines the equivalent open-loop process for each loop i with all other loops closed can be given as:

$$g_{ii}^{eq}(s) = g_{ii}(s) - \sum_{j=1}^{n} \frac{g_{ij}(s)c_j(s)g_{ji}(s)}{1 + g_{jj}(s)c_j(s)} + \frac{g_{ii}(s)c_i(s)g_{ii}(s)}{1 + g_{ii}(s)c_i(s)}. \tag{6}$$

4. CONTROLLER DESIGN BY LINEAR PROGRAMMING

Consider the SISO controller design method of Karimi et al. (2007), which computes controller parameters by a convex optimization problem that minimizes the disturbance effects subject to stability margins constraints. The proposed method in this article modifies the original SISO method to also take loops interactions into account in the controller design by Gershgorin bands and EOPs.

4.1 The Cost Function

Load disturbance rejection can be expressed in terms of the integrated absolute error due to a unit step load disturbance at process input,

$$IAE = \int_0^{\infty} |e(t)|\mathrm{d}t. \tag{7}$$

This criterion is difficult to deal with analytically because the evaluation requires the computation of time functions. The integrated error defined by

$$IE = \int_0^{\infty} e(t)\mathrm{d}t, \tag{8}$$

is more convenient. In Åström et al. (2006) it is shown that $IE = 1/k_i$. Thus, minimizing IE is equivalent to maximize k_i. If the system is well damped, the quantities of IE and IAE are approximately the same.

4.2 The Loop Transfer Function Constraint

Consider a straight line r_n which crosses the negative real axis between 0 and -1 with a distance ℓ from the critical point. The angle between r_n and the real axis is α which is a value between 0° and 90°, see Figure 3. This straight line divides the complex plane into two regions. The purpose