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Simulation studies of inverted pendulum based on PID controllers

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ABSTRACT

The inverted pendulum problem is one of the most important problems in control theory and has been studied excessively in control literatures. When a control system have more than two PID controllers, the adjustment of PID parameters is not an easy problem. In this paper, PID controllers are applied to the stabilization and tracking control of three types of inverted pendulum. The way how to design the PID controllers is given step by step in this paper. Simulation results prove that the way to design of PID controllers is very simple and effective. The system design not only can realize stabilization and tracking control of three types of inverted pendulum, but also have robustness to outer large and fast disturbances.

1. Introduction

The inverted pendulum problem is one of the most important problems in control theory and has been studied excessively in control literatures. It is well established benchmark problem that provides many challenging problems to control design. The system is nonlinear, unstable, nonminimum phase and underactuated. Because of their nonlinear nature pendulums have maintained their usefulness and they are now used to illustrate many of the ideas emerging in the field of non-linear control [1]. The challenges of control made the inverted pendulum systems a classic tools in control laboratories.

According to control purposes of inverted pendulum, the control of inverted pendulum can be divided into three aspects. The first aspect that is widely researched is the swing-up control of inverted pendulum [2,3]. The second aspect is the stabilization of the inverted pendulum [4–6]. The third aspect is tracking control of the inverted pendulum [7,8]. In practice, stabilization and tracking control is more useful for application.

It is rather surprising that virtually almost all the technical literature refers to the inverted pendulum with one freedom. Only recently there are a few references dealing with the inverted pendulum with two or three degrees of freedom [7–10]. In this paper, we give three types of inverted pendulum. The first type is the most customary inverted pendulum that can only move in x horizontal direction. We call this type inverted pendulum as x inverted pendulum. The second type of inverted pendulum can move in the x–y horizontal plane [7–9]. We call this type inverted pendulum as x–y inverted pendulum. The third type of inverted pendulum can move in the x–z horizontal and vertical plane which is first proposed by Maravall [11,12]. We call this type inverted pendulum as x–z inverted pendulum. The models of three types of inverted pendulum are analyzed in details. The relations between the three types of inverted pendulum are also given in this paper.

Ref. [13] applied interval type-2 fuzzy sliding-mode controller in the inverted pendulum. But the model of the inverted pendulum did not consider the dynamic of the cart of the inverted pendulum. Ref. [14] applied coupled sliding-mode control to orbital stabilization of inverted pendulum systems. In Ref. [15], the proposed methodology, which performs swing up and control simultaneously, uses elements from input–output linearization, energy control, and singular perturbation theory. In

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Ref. [16], the authors presented a design of an optimized fuzzy cascade controller based on hierarchical fair competition-based genetic algorithms (HFCGA) for a rotary inverted pendulum system.

Although a lot of control algorithm are researched in the systems control design, PID controller is still the most widely used controller structure in the realization of a control system. The overwhelming advantages of PID controller, which have greatly contributed to its wide acceptance, are its simplicity and sufficient ability to solve many practical control problems. To the present, there are lots of control strategies applied to the inverted pendulum control. But there are very few reference about PID control in inverted pendulum control. When a control system have more than two PID controllers, the adjustment of PID parameters is not a easy problem. In this paper, the design of the PID controllers is given in details. This scheme makes the inverted pendulum control design very simple based on PID controllers.

The organization of this paper is as follows. Section 2 will introduce the structure and models of three types of inverted pendulum. The relations between three types of inverted pendulum are also analyzed. In Section 3, we will give the design procedure of the PID controllers for every type of inverted pendulum. Simulation results of three types of inverted pendulum controlled by PID controllers are shown in different control conditions. Section 4 gives the conclusions of the paper.

2. Structure and models of three types of inverted pendulum

2.1. Structure and model of x inverted pendulum

The x inverted pendulum on a pivot driven by horizontal control force is shown in Fig. 1(a). In Fig. 1(a), the control action is based on the horizontal displacements of the pivot.

The total kinetic energy and potential energy of the x inverted pendulum are

$$K = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}_p{}^2 + \dot{z}_p{}^2\right), \quad P = mgz_p, \tag{1}$$

where $x_p = x + l\sin\theta$, $z_p = z + l\cos\theta$, l is the distance from the pivot to the mass center of the pendulum, M, m, are the mass of the pivot and the pendulum respectively, (x,z) is the position of the pivot in the xoz coordinate, $(\dot{x},\dot{z})$ is the speed in the xoz coordinate, (x_p,z_p) is the position in the $x'o'z'$ coordinate, $(\dot{x}_p,\dot{z}_p)$ is the speed in the $x'o'z'$ coordinate, g is the acceleration constant due to gravity. We assume that the inertia of the pendulum is negligible. L is the system Lagrangian of the x inverted pendulum and is defined as $L = K - P$.

The Lagrange's equations of the x inverted pendulum are

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x, \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0, \end{cases} \tag{2}$$

where F_x is the horizontal control force. According to Eqs. (1), (2) the Lagrange's equations of the x inverted pendulum can be expressed as

$$\begin{cases} (M+m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 = F_x, \\ \cos\theta\ddot{x} + l\ddot{\theta} - g\sin\theta = 0, \end{cases} \tag{3}$$

where $-0.5 \leqslant x \leqslant 0.5$. According to Eq. (3), the state equations of the x inverted pendulum can be expressed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{-mg\cos x_3 \sin x_3 + ml\sin x_3 x_4^2 + F_x}{M + m\sin^2 x_3} + d_1, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = \frac{-ml\cos x_3 \sin x_3 x_4^2 - \cos x_3 F_x + (M+m)g\sin x_3}{Ml + ml\sin^2 x_3} + d_2, \end{cases} \tag{4}$$

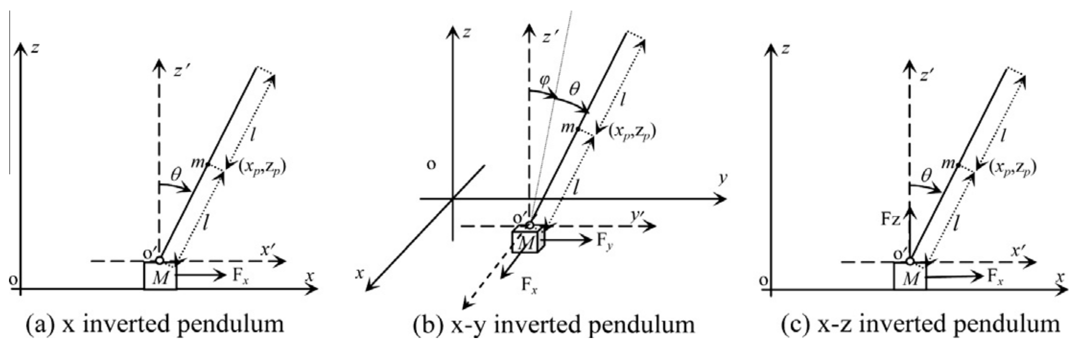

Fig. 1. Structure of three types of inverted pendulum.