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New results on the synthesis of FO-PID controllers

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ABSTRACT

In this paper a new procedure that allows to define the parameters of fractional order $PI^{\lambda}D^{\mu}$ controller, designed to stabilize a first-order plant with delay-time, is proposed. Using a version of the Hermite–Biehler theorem applicable to quasipolynomials, the complete set of stabilizing $PI^{\lambda}D^{\mu}$ parameters is determined. The widespread industrial use of PID controllers and the potentiality of their non-integer order representation justifies a timely interest to $PI^{\lambda}D^{\mu}$ tuning techniques.

1. Introduction

In the last years a renewed interest has been devoted to fractional order systems in the area of automatic control.

It is possible to apply non-integer order systems for control purposes as in [1–4], and in robotic [5], while different practical controller implementations have been suggested in [6–8]. The authors, in [9,10] proposed an analog implementation of the non-integer order integrator based on Field Programmable Analog Arrays (FPAAs), able to implement $PI^{\lambda}D^{\mu}$ controller.

A lot of interest is equally devoted to $PI^{\lambda}D^{\mu}$ parameters tuning, see for example [11–14].

In the world a lot of control systems are operated by industrial PID controllers. Thanks to the widespread industrial use of PID controllers, even a small improvement in PID features, achieved by using $PI^{\lambda}D^{\mu}$, could have a relevant impact.

During the last decades, numerous methods have been developed for setting the parameters of P, PI, and PID controllers. Some of these methods are based on characterizing the dynamic response of the plant to be controlled with a first-order model with time delay. It is interesting to note that even though most of these tuning techniques provide satisfactory results, the set of all stabilizing PID controllers for these first-order models with time delay remains unknown.

In [15] a generalization of the Hermite–Biehler theorem was derived and then used to compute the set of all stabilizing PID controllers for a given linear, time invariant plant, described by a rational transfer function. The approach developed in [15] constitutes the first attempt to find a characterization of all stabilizing PID controllers for a given plant. However, the synthesis results presented in that reference cannot be applied directly to plants containing time delays since they were obtained for plants described by rational transfer functions.

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Plants with time delays give rise to characteristic equations containing quasipolynomials.

Our approach in this paper will be to make use of a version of the Hermite–Biehler theorem applicable to quasipolynomials. Such a result was derived by Pontryagin in [16,17]. These references represent the substantial progress made by several researchers in studying the stability of time delay systems with up to two variable parameters. In [18] the problem of characterizing the set of PID parameters that stabilize a given first-order plant with time delay involves three adjustable parameters. The aim of this paper is to provide a complete solution to the problem of characterizing the set of $PI^{\lambda}D^{\mu}$ parameters that stabilize a given first-order plant with time delay using a version of the Hermite–Biehler theorem applicable to quasipolynomials in the area of Fractional Order Systems (FOS).

2. Problem characterization

Systems with step responses like the one shown in Fig. 1 are commonly modelled as first-order processes with a time delay, and can be mathematically described by:

$$G(s) = \frac{k}{1+Ts}e^{-Ls} \tag{1}$$

where k represents the steady-state gain of the plant, L represents the time delay, and T represents the time constant of the plant.

Consider now the feedback control system shown in Fig. 2 where u is the command signal, y is the output of the plant, $G(s)$, given by (1), is the plant to be controlled, and C(s) is the controller. In the paper the authors focus on the case when the controller is a fractional order $PI^{\lambda}D^{\mu}$, i.e.,

$$C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \tag{2}$$

The aim of the paper is to determine, after fixing the fractional orders of the integrative and derivative actions, the set of controller parameters (k_p, k_i, k_d) for which the closed-loop system is stable.

3. Analyzing systems with time delays

Many problems in process control engineering involve time delays. These time delays lead to dynamic models with characteristic equations of the form

$$\delta(s) = d(s) + e^{-sT_1}n_1(s) + e^{-sT_2}n_2(s) + \cdots + e^{-sT_m}n_m(s) \tag{3}$$

where $d(s)$, $n_i(s)$ for $i = 1, 2, \ldots, m$ are polynomials with real coefficients.

Characteristic equations of this form are known as quasipolynomials. It can be shown that the so called Hermite–Biehler theorem for Hurwitz polynomials (see [19,20]), does not carry over to arbitrary functions $f(s)$ of the complex variable s. Pon-

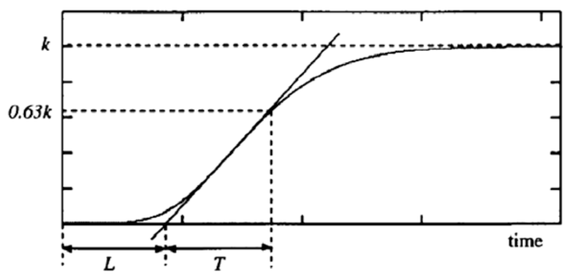

Fig. 1. Open-loop step response.

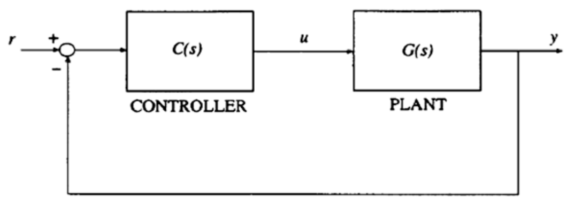

Fig. 2. Feedback control system.