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A μ-analysis approach to power system stability robustness evaluation

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Abstract

A general approach based on structured singular value (SSV) theory is proposed for the analysis of robust stability of large power systems with parameter variations. SSV theory is utilized to find a structured-based parametric uncertainty description of the system that explicitly address and treat the effect of multiple, interrelated uncertain parameters on system dynamic behavior. Techniques to handle parametric uncertainties are given and methods to generate linear fractional transformation based uncertainty descriptions for the model and its associated uncertainty are discussed. Attention is focused on the study of uncertainties in the nominal system representation arising from two different sources, namely variations in the system operating conditions and uncertainties in the structure of the power system. The performance robustness of the proposed method is verified through simulation studies on a 6-area, 377-machine practical system. In particular, the developed technique is used to estimate the combined effect of variations in the level of power transfer across a critical system interconnection, and variations in the interconnecting tie-line reactance on the stability of critical inter-area modes. Several case studies are presented in which both conventional eigenanalysis techniques and SSV-theory are used to determine robust stability margins.

Keywords: Structured singular value; Robustness analysis; Parametric uncertainty; Small signal analysis

1. Introduction

Uncertainty modeling is increasingly finding use in the description and forecasting of uncertain behavior in robust flight control, and in various engineering fields [1–4]. The ability of a closed-loop control system to retain stability in the presence of parameter variations is an important aspect that is not guaranteed with conventional design techniques.

Uncertainties in the power system model can be unstructured, i.e. representative of unmodeled system dynamics [1,2], or structured associated with slow variations in power system parameters, such as loading and other changes in operating conditions [4–6] and can result in conservative assessment of system stability.

Characterization of system uncertainty, however, is a challenging problem. During the past two decades there has been significant progress in the development of analysis and design techniques for systems with parametric uncertainties and unmodeled dynamics, using SSV theory. The most general and accurate means of analyzing and characterizing the effect of system uncertainty on robust performance and stability, is the structured singular value μ-framework developed by Doyle and other researchers [7–9]. This approach allows for the precise measurement of the effects of changes in operating conditions and uncertainty in model parameters on stability robustness and performance robustness.

Early systematic applications of these methods concentrated on interpreting system uncertainty and were restricted to systems with small dimensions [4,5]. In Ref. [6], the SSV framework was used to determine robust stability margins of a power system with respect to perturbations on operating conditions. Recent work has focused on the analysis of robust stability and control performance in power systems subject to changes in both, operating conditions and uncertainty in model parameters [6,10].

This paper examines the application of SSV theory for robust stability assessment of large power systems with structured parametric uncertainties. An SSV-based uncertainty model is used to find a representation of the power system that incorporates multiple uncertainties. In this approach, variations in system operating conditions and parameters are represented

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as structured uncertainties and included in the nominal power system representation.

The stability of the resulting closed-loop power-system model is then evaluated in terms of its robustness in the presence of these uncertainties. Specifically, attention is focused on the study of simultaneous structured uncertainties in the nominal system representation arising from two distinct sources: variations in the amount of power transferred through critical system interconnections, and variations in the interconnecting tie-line reactance. SSV analysis is used to estimate both the limiting loading conditions and the uncertain elements that dominate the robust performance. Such an accurate prediction capability would significantly reduce the amount of computational effort required for assessing the maximum change in expected operating conditions to make the system unstable. Moreover, the proposed procedures are general and can be easily incorporated into existing small signal stability software.

The performance robustness of the proposed method is verified through simulation studies on a large-scale power system. The validation is based on comparison with results from repeated eigenvalue studies using a commercially available small signal stability program. Simulation results indicate that the proposed method is both accurate and flexible and can accurately be used to assess robust stability in large power systems.

2. Power system model in the SSV framework

2.1. The perturbed state-space system

Consider a general uncertain system represented by the state-space realization

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} \tag{1}$$

where $\mathbf{p} = p_1, p_2, \ldots, p_m$ is the set of uncertain parameters which are assumed to vary within some practical limits $p_k^{\min} \le p_k \le p_k^{\max}$ about the nominal value.

Several approaches to transform the model (1) into a convenient form for μ-analysis have been proposed in the literature, including symbolic linearization methods and numerical techniques. Traditional methods aimed at studying robust stability of uncertain process are based on the local linearization of the model equations and subsequently applying theorems developed for linear systems. Assuming that the elements a_{ij} of the state matrix $\mathbf{A}(\mathbf{p})$ are analytic functions of the parameters, p_k ($k = 1, \ldots, m$), and that they can be represented accurately by their Taylor series expansions, we obtain

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + \sum_{k=1}^{m} \mathbf{A}_k \Delta p_k = \mathbf{A}(p_k = 0) + \left.\frac{\partial \mathbf{A}}{\partial p_k}\right|_{p_k^{\mathrm{o}}=0} \Delta p_k + \cdots \tag{2}$$

where the partial derivatives in the above expressions can be obtained analytically from knowledge of the system structure, or numerically using finite differences. Here, $\mathbf{A}_0$ represents the nominal system, whilst the matrices $\mathbf{A}_k$, $k = 1, \ldots, m$, represent deviations from the nominal conditions which explicitly depend on the physical real value parameter, p_k.

The derived analytical formulation can then be converted to a form suitable for robust stability analysis. With this approach, it becomes possible to study robust stability properties in an analytical setting where repeated numerical linearizations are conducted to uncover the robustness properties in the state-space as function of system parameters.

In the sequel, we discuss techniques to recast this formulation into the real-μ analysis framework.

2.2. Parametric uncertainty modeling

A key issue in applying robustness theory is allowing for the role of parameter variations whose exact values are unknown but which are known to lie between some minimum and maximum values. In robust stability theory, uncertainty is represented by a scalar perturbation to the nominal model. Thus, for instance, if a parameter p_k, has a nominal value p_k^{o}, but an uncertainty of $\pm r$, then the parametric uncertainty can be expressed as a parameter set of the form

$$p_k = \bar{p}_k(1 + r_k\delta_k), \quad i = 1, \ldots, m \tag{3}$$

in which

$$\bar{p}_k = \frac{(p_k^{\min} + p_k^{\max})}{2}, \qquad r_k = \frac{p_k^{\max} - p_k^{\min}}{p_k^{\min} + p_k^{\max}},$$

$$p_k \in [p_k^{\min}, p_k^{\max}]$$

where $\bar{p}_k$ is the mean parametric value and r_k is the relative uncertainty in the parameter; δ_k is a real scalar such that $-1 \le \delta_k \le 1$. With this notation in mind, we note that $p_k = p_k^{\max}$ for $\delta_k = 1$, and $p_k = p_k^{\min}$ for $\delta_k = -1$. From the previous definition it follows that the uncertain system representation can be written in terms of the nominal and perturbed parameters as

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} = \left[\mathbf{A}_0(\mathbf{p}^{\mathrm{o}}) + \sum_{k=1}^{m} \mathbf{A}_k \delta_{p_k}\right]\mathbf{x} = [\mathbf{A}_0(\mathbf{p}^{\mathrm{o}}) + \mathbf{A}(\delta)]\mathbf{x} \tag{4}$$

where matrix $\mathbf{A}_0$ represents the nominal plant dynamics, $\mathbf{p}^{\mathrm{o}} = p_{k_1}^{\mathrm{o}}, p_{k_2}^{\mathrm{o}}, \ldots, p_{k_m}^{\mathrm{o}}$ is the uncertain parameter vector at the linearization point, and matrices $\mathbf{A}_k$, $k = 1, \ldots, m$ describe deviations from the nominal system; the scalar uncertainty, δ_{p_k}, represents an unknown varying coefficient whose values belong to an uncertainty interval $\delta_{p_k}^{\min} \le \delta_{p_k} \le \delta_{p_k}^{\max}$.

Referring to Eq. (3) it is to be noted that the local parametric uncertainty can be expressed by a upper linear fractional transformation (LFT) defined by [12]

$$p_k = F_{\mathrm{u}}(\mathbf{\Gamma}, \mathbf{\Delta}) = F_{\mathrm{u}}\left(\begin{bmatrix} 0 & r_k\bar{p}_k \\ 1 & \bar{p}_k \end{bmatrix}, \delta_{p_k}\right)$$
$$= \mathbf{\Gamma}_{22} + \mathbf{\Gamma}_{21}(\mathbf{I} - \mathbf{\Delta}\mathbf{\Gamma}_{11})^{-1}\mathbf{\Delta}\mathbf{\Gamma}_{12} \tag{5}$$

where $\Delta = \delta_{p_k}$, and

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} = \begin{bmatrix} 0 & r_k\bar{p}_k \\ 1 & \bar{p}_k \end{bmatrix}$$