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Stabilization of control loops consisting of FOPDT process and parameter-dependent PID controller

Ali Madady*, Hamid-Reza Reza-Alikhani

Department of Electrical Engineering, Tafresh University, Tehran Ave., Tafresh, Iran

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ABSTRACT

In this paper, problem of stability analysis of the control loops consisting of first-order plus dead time (FOPDT) processes and proportional-integrative-derivative (PID) controllers is studied, where the controller coefficients are functions of one or more independent parameters. An effective procedure is presented to determine a stability region in the independent parameters space. This method does not require complex numerical calculations such as solving nonlinear equations. It is based on usage of a two-valued indicator function and by using that, a stability region is easily determined. In order to clarify that, why the stability region needs to be specified in the "independent parameters space" an optimal method is given to design the PID controller for the FOPDT processes, as an instance. In this optimal method the controller coefficients are obtained as the functions of a free parameter, where this parameter needs to be chosen by the designer such that it should be near to the maximum operating frequency of the system, besides on the other hand the closed-loop system to be stable. In the end, two illustrative examples are given in order to show the usefulness and effectiveness of the proposed method, and to compare the obtained stability regions with the whole stability regions.

1. Introduction

Dynamics of many industrial processes can be well modeled by a stable first-order plus dead time (FOPDT) transfer function as follows [1–3]:

$$G_P(s) = \frac{ke^{-Ls}}{1+Ts} \tag{1}$$

where k represents the steady-state gain, $L>0$ is the time delay, and $T>0$ represents the time constant of the plant. Without loss of generality, it is assumed that $k>0$.

Model (1) has been used quite often, because of its simplicity and its capability to obtain the fundamental dynamics of many industrial processes [3]. In order to identify the parameters k, T and L, a few identification methods have been presented. For the on-line identification of the FOPDT processes an exact method based on ideal relay was given in [4]. A closed-form analytical expression was obtained for the parameters of the FOPTD systems from knowledge of two relative extremum in the transient response to two different finite-duration pulse inputs [5]. In order to identify the FOPTD systems a simple graphical method that uses the integral of pulse response was proposed by [6]. An indirect identification scheme was developed in [7] for determining the parameters of the FOPTD and the second-order plus dead time (SOPTD) models from their step response. Using behavior of an objective function in a neighborhood of the actual delay, an on-line method was presented to estimate the delay for stable, unstable and integrating systems under step input, which is as well applicable for FOPTD systems [8].

In many control applications such as industrial control systems, proportional-integrative-derivative (PID) controllers are for sure the most used methods. The main reason is that, PID control has a simple structure with three meaningful parameters, which makes it easier to understand than the majority of other advanced control techniques by the control engineers. Furthermore, from the performance point of view, PID control is generally quite acceptable in most cases [1,2,9]. For this reason these controllers are widely used in controlling the FOPDT processes, and many different tuning procedures have been proposed for them.

Various methods have been put into practice in order to tune PID controllers based on the model (1), and the significant ones investigated in [1,2]. A complete review of well known formulas can be found in [10,11]. New approaches are based on Hermite–Biehler Theorem [12], relay feedback [13], internal model control (IMC) [9,14], partial compensation [15], neural networks [16], modern optimization [17], event-based degrees of freedom [18], third quadrant Nyquist points [19], as well as minimizing the integrated absolute error with a constraint on the maximum sensitivity [20]. A novel design synthesis was provided for PID controllers to get two

* Corresponding author.
E-mail addresses: drmadady@yahoo.com (A. Madady), alikhani.hamid@gmail.com (H.-R. Reza-Alikhani).

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pre-specifications (phase margin and gain crossover frequency) and flat phase tuning constraint for the FOPDT systems [21], so that the obtained controller is robust not only to the uncertainty of the plant steady-state gain, but also to the entire variation of the controller coefficients. Recently a robust tuning method, which is based on the usage of a model reference optimization procedure with servo and regulatory target closed-loop transfer functions, was presented for two degree of freedom proportional integral (PI) controllers for FOPTD and SOPTD processes [22].

However, stability is the most important requirement in any control loop design. For this reason in a control system, the coefficients of the controller must be chosen and adjusted so that the closed-loop system to be stable initially. That is why to determine the stability region in the controller coefficients space has a high significant. For this, stability analysis of the control loops consisting of a FOPDT process and a PID controller is studied in [23,24]. The approach of [23,24] is based on a version of the Hermite–Biehler Theorem, where the complete set of stabilizing PID coefficients is determined. Latter the same results are obtained in a more simple way by means of the classical Nyquist's stability criterion in [25]. Recently by exploiting the same version of the Hermite–Biehler Theorem, which is used by [23,24], the set of stabilizing parameters of a PID controller was determined for an integrator plus dead-time process [26].

If the controller coefficients are not directly and independently adjustable and they are functions of one or many independent parameters so that by adjusting these parameters the controller coefficients will be adjusted, then the problem of stabilization is to determine the stability region in the "independent parameters space" instead of determining this region in the controller coefficients space. For example, let assume in a closed-loop system the controller, which is a PID type electronic circuit, has an adjustable resistor R and an adjustable capacitor C, with the rest of the components to be constant. That is, the controller coefficients are functions of R and C as the following:

$$k_P = g_P(R, C), \quad k_I = g_I(R, C), \quad k_D = g_D(R, C)$$

For this case, instead of the determining of the stability region in the three-dimensional space (k_p, k_1, k_d), it is needed the stability region to be determined directly in the two-dimensional space (R, C) in order to know for what values of R and C the closed-loop system becomes stable.

Studying and solving this problem for the control-loops consisting of FOPDT processes and PID controllers is the aim and motivation of this paper. As in these cases, where the controller coefficients are not directly and independently adjustable and are the functions of some independent parameters, the results of [23–25] are not applicable. Since the closed-form solution of a complex nonlinear equation is needed and it is not possible to obtain this solution.

The paper is organized as follows. Section 2 gives the necessary preliminaries. In Section 3, the underlying problem is described. The proposed solution method is presented in Section 4. As an instance, an optimal method is developed to design the PID controller for FOPDT processes in Section 5. Illustrative examples are given by Section 6. The last Section 7 is devoted to the conclusions.

2. Preliminaries

Consider plant (1) which is controlled by a PID controller as unity output feedback configuration, which is shown in Fig. 1. Where the transfer function of the controller is:

$$G_C(s) = k_P + \frac{k_I}{s} + k_D s \tag{2}$$

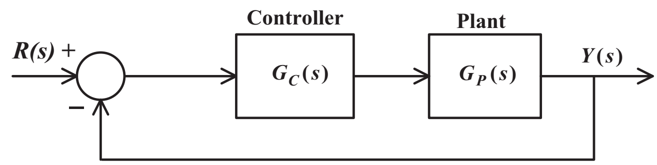

Fig. 1. The unity output feedback control system.

The stability of the closed-loop system of Fig. 1 is studied in [23,24] using a version of the Hermite–Biehler Theorem, and the complete set of stabilizing PID parameters is determined. The summary of the obtained results in [23] is reviewed here.

The range of k_p values for which the closed-loop system is stable is given by [23]:

$$-\frac{1}{k} < k_P < k_u \tag{3}$$

where

$$k_u = \frac{1}{k}\left[\frac{\alpha_1}{\mu}\sin(\alpha_1) - \cos(\alpha_1)\right], \quad \mu \triangleq \frac{L}{T} \tag{4}$$

and α_1 is the solution of the following equation in the interval (0, π):

$$\tan(\alpha) = -\frac{\alpha}{1+\mu} \tag{5}$$

For k_p values outside the range given in (3), there are no stabilizing PID controllers. The complete stabilizing region is given by (see Fig. 2) [23]:

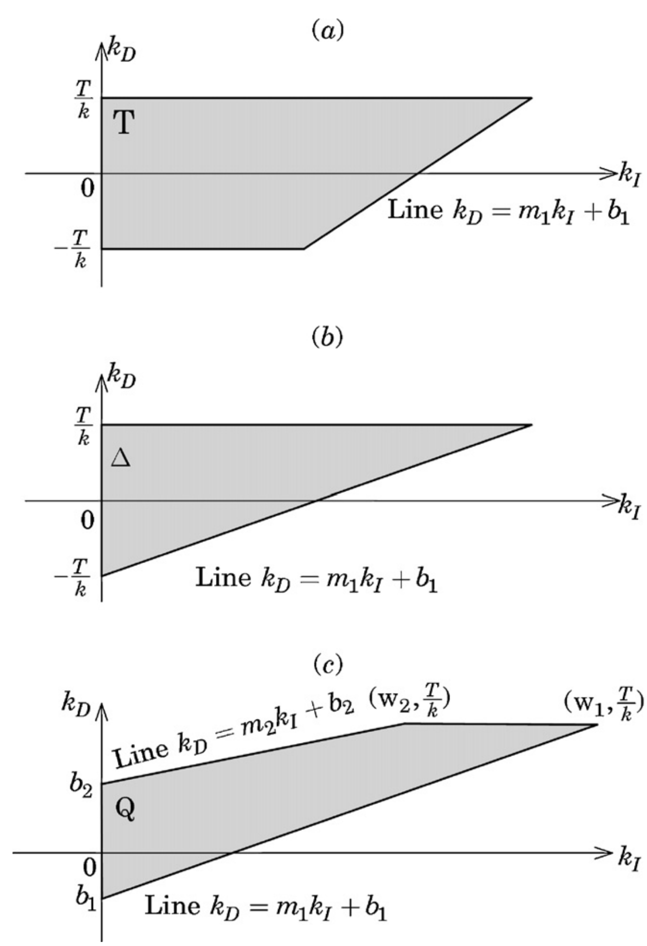

Fig. 2. The stabilizing region of (k_I, k_D) for: (a) $-1/k < k_P < 1/k$, (b) $k_P = 1/k$, and (c) $1/k < k_P < k_u$.