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Reliable decentralized PID controller synthesis for two-channel MIMO processes[☆]

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ABSTRACT

Reliable stabilization and regulation of two-channel decentralized multi-input multi-output (MIMO) control systems is considered. The system has integral-action due to using proportional + integral + derivative (PID) controllers. Closed-loop stability and asymptotic tracking of step-input references are achieved at each output channel when all controllers are operational. Stability is maintained when one of the controllers fails completely and is set to zero. Controller synthesis procedures are proposed for stable MIMO plants and for several unstable MIMO plant classes that admit PID controllers. These synthesis procedures are applied to various examples of process systems to illustrate the design methodology.

1. Introduction

A stabilizing controller synthesis method is developed for linear, time-invariant (LTI), multi-input multi-output (MIMO) systems using a two-channel decentralized controller structure, with the objectives of decentralized closed-loop stabilization, reliable stability in case of complete failure of one of the two channels, and integral-action achieved with low-order simple PID controllers.

The decentralized controller structure has advantages although it restricts the stabilizing controller class. Fully decentralized control designs can be difficult also because of the interaction between the control loops. This introduces the problem of input–output pairing, to be decided in the first stage of the design before controller tuning. A method used to measure interactions and assess appropriate pairing is the relative gain array (RGA) (Campo & Morari, 1994). It is assumed here that the input–output pairing of the decentralized structure is already completed and the given plant model is partitioned into two MIMO channels *a priori*. An important control design requirement is reliability of the system's closed-loop stability against complete failure of certain channels. Reliable designs were considered under full-feedback and decentralized controller structures in Braatz, Morari, and Skogestad (1994), Gündeş and Kabuli (2001), Siljak (1980) and Tan, Siljak, and Ikeda (1992). In reliable control systems, when sensor and/or actuator failures occur and controllers in failed channels are taken out of service, the remaining controllers maintain closed-loop stability of the entire system. The completely different approach of fault tolerant control, based on first defining and storing all feasible controllers, guarantees stability by using a switching strategy among these controllers depending on failures (Seron, Zhuo, De Dona, & Martinez, 2008). Reliable stabilization requires no switching or re-tuning of controllers. An important performance objective is asymptotic tracking of constant reference inputs with zero steady-state error, achieved by designing controllers with integral-action. The simplest integral-action controllers are in proportional + integral + derivative (PID) form (Goodwin, Graebe, & Salgado, 2001). Although PID controllers are desirable due to easy implementation and tuning, their simplicity presents a major restriction that they can control only certain plants.

The problem studied here has several layers of difficulty due to the restricted decentralized structure of the controller, the requirement of closed-loop stability when one controller is taken out, and the restrictions in the class of (unstable) processes that can be stabilized using PID controllers. Achieving reliable closed-loop stability with either one of the controllers subject to failure is more demanding on the design than expecting that stability is maintained when a pre-specified one of the two controllers may fail. It is assumed that the failure of the controller C_j, $j = 1, 2$,

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is recognized and the failed controller is taken out of service (with its states reset to zero). This catastrophic failure is modeled by setting $C_j = 0$. A design that is reliable against either failure is called *fully reliable*; if only one specific controller may fail, then the control design is called *partially reliable*. These and other definitions are provided in Section 2, where the problem considered is formally stated. Section 3 is devoted to MIMO processes that are open-loop stable. Although both partially and fully reliable decentralized controllers exist for stable plants, the requirement of PID controllers and integral-action imposes additional conditions on the plants. These conditions and a reliable decentralized PID controller synthesis procedure are given in Section 3.1. For the stable plant case in Section 3, any other PID tuning method can be used to design C_2 that stabilizes one sub-block of the plant; for a fully reliable design, C_1 simultaneously stabilizes two systems related to the plant. Each of these blocks to be stabilized are MIMO systems if each channel has multiple inputs and outputs. There are no established PID tuning methods achieving simultaneous stabilization of two systems applicable in the MIMO setting. The synthesis methods proposed in this section are illustrated by examples. For certain PID stabilizable unstable plant classes (Gündeş & Özgüler, 2007), Section 4 investigates existence conditions for reliable decentralized controllers and proposes controller synthesis procedures. Partially reliable PID controller designs are illustrated for two examples.

The designs proposed here achieve closed-loop stability and asymptotic tracking of step-input references with zero steady-state error when all channels are operational, and maintain closed-loop stability of the overall system when either channel fails, with integral-action still present in the channel that remains active. The proposed designs also achieve asymptotic rejection of output disturbances since this is mathematically equivalent to the tracking problem. The proposed controllers also achieve robust closed-loop stability under sufficiently small additive or multiplicative plant uncertainty. The synthesis procedure for each plant class considered here allows freedom in choosing many of the design parameters. These parameters may be chosen to optimize the response in case of other performance specifications. Since the only goal here is reliable regulation, only stability and asymptotic tracking of constant inputs are emphasized and other performance objectives are not specified.

Notation. Let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$ denote complex, real, positive real numbers; $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$ is the extended closed right-half plane; I_n is the $n \times n$ identity matrix; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S}$ is the stable subset with no $\mathcal{U}$-poles; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$. A square matrix $M \in \mathcal{M}(\mathbf{S})$ is called unimodular iff $M^{-1} \in \mathcal{M}(\mathbf{S})$. The H_∞-norm of $M(s) \in \mathcal{M}(\mathbf{S})$ is $\|M\| := \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$; $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$. Wherever this causes no confusion, (s) in transfer functions such as $G(s)$ is dropped. We use coprime factorizations over $\mathbf{S}$. We abbreviate right-coprime (RC) and left-coprime (LC).

2. Problem statement and preliminaries

Consider the LTI decentralized feedback system $Sys(G, C_D)$ with two MIMO channels as in Fig. 1. The feedback system is well-posed; the plant and controller have no hidden-modes associated with eigenvalues in the unstable region $\mathcal{U}$. The plant $G \in \mathbf{R_p}^{r\times m}$ and the decentralized controller $C_D \in \mathbf{R_p}^{m\times r}$ are partitioned as:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad G_{ij} \in \mathbf{R_p}^{r_i\times m_j}, i, j = 1, 2, \tag{1}$$

$$C_D = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \text{diag}\,[\,C_1\,,\; C_2\,], \quad C_j \in \mathbf{R_p}^{m_j\times r_j}. \tag{2}$$

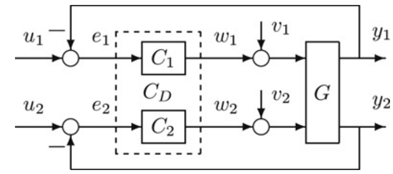

Fig. 1. The two-channel decentralized system $Sys(G, C_D)$.

Let $m := m_1 + m_2$, $r := r_1 + r_2$. We assume throughout that rank $G(s) = r$ and rank $G_{jj}(s) = r_j$ for $j = 1, 2$. Let

$$u := \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad v := \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad e := \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$
$$w := \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad y := \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

denote the input and output vectors. Then the closed-loop transfer function H_{cl} from (u, v) to (w, y) is:

$$H_{cl} = \begin{bmatrix} C_D(I + GC_D)^{-1} & -C_D(I + GC_D)^{-1}G \\ GC_D(I + GC_D)^{-1} & (I + GC_D)^{-1}G \end{bmatrix}. \tag{3}$$

We have three main goals in developing a systematic synthesis method: **(1)** Design the decentralized stabilizing controller C_D so that the closed-loop system $Sys(G, C_D)$ achieves asymptotic tracking of step-input references with zero steady-state error. **(2)** The design should be reliable so that closed-loop stability is achieved if both channels are operational, and is still maintained even if one of the controllers C_1 or C_2 fails completely. The failure of C_j is recognized and the failed controller is taken out of service (with its states reset to zero). This catastrophic failure is modeled by setting the failed controller's transfer-matrix equal to zero. With $C_1 = 0$, the system is called $Sys(G, 0, C_2)$; with $C_2 = 0$, the system is called $Sys(G, C_1, 0)$. The failed channel does not achieve asymptotic tracking with zero steady-state error. **(3)** The controller order should be restricted; reliable decentralized stabilization should be achieved using PID controllers, where $C_j \in \mathbf{R_p}^{m_j\times r_j}$, $j = 1, 2$, should be in the proper PID controller form (Goodwin et al., 2001):

$$C_j = K_{Pj} + \frac{1}{s}K_{Ij} + \frac{s}{\tau_j s + 1}K_{Dj}, \quad j = 1, 2 \tag{4}$$

where $K_{Pj}, K_{Ij}, K_{Dj} \in \mathbb{R}^{m_j\times r_j}$ are called the proportional, integral, and derivative constants, respectively, and $\tau_j \in \mathbb{R}_+$, $j = 1, 2$. The integral-action in C_j is present when $K_{Ij} \neq 0$. Subsets of PID controllers are obtained by setting one or two of the three constants equal to zero; (4) becomes a PI-controller when $K_{Dj} = 0$ and an I-controller when $K_{Pj} = K_{Dj} = 0$.

Let $G = Y^{-1}X$ be a left-coprime-factorization (LCF) of $G \in \mathbf{R_p}^{r\times m}$, where $X \in \mathbf{S}^{r\times m}$, $Y \in \mathbf{S}^{r\times r}$, $\det Y(\infty) \neq 0$. Let $C_D = N_c D_c^{-1}$ be a right-coprime-factorization (RCF) of $C_D \in \mathbf{R_p}^{m\times r}$, where $N_c \in \mathbf{S}^{m\times r}$, $D_c \in \mathbf{S}^{r\times r}$, $\det D_c(\infty) \neq 0$. Let the (input-error) transfer-function from u to e be denoted by H_{eu} and let the (input–output) transfer-function from u to y be denoted by H_{yu}; then $H_{eu} = (I_r + GC_D)^{-1} = I_r - GC_D(I_r + GC_D)^{-1} =: I_r - GH_{wu} =: I_r - H_{yu}$.

Definition 1. **(a)** The system $Sys(G, C_1, C_2)$ is *stable* if the closed-loop transfer-function H_{cl} from (u, v) to (w, y) is stable. **(b)** The system $Sys(G, 0, C_2)$ is *stable* if with $C_1 = 0$, the closed-loop transfer-function H_2 from (u_2, v) to (w_2, y) is stable. **(c)** The system $Sys(G, C_1, 0)$ is *stable* if with $C_2 = 0$, the closed-loop transfer-function H_1 from (u_1, v) to (w_1, y) is stable. **(d)** The stable system $Sys(G, C_1, C_2)$ has *integral-action* if $H_{eu}(0) = 0$, i.e., H_{eu} has blocking-zeros at $s = 0$. **(e)** The decentralized stabilizing controller $C_D = \text{diag}\,[\,C_1, C_2\,]$ is an *integral-action controller* if