Journal of Process Control 21 (2011) 620–626

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Classification of dynamic processes and PID controller tuning in a parameter plane

Tomislav B. Šekara, Miroslav R. Mataušek*

Faculty of Electrical Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, Belgrade 11000, Serbia

ARTICLE INFO

Article history:
Received 21 June 2010
Received in revised form 6 December 2010
Accepted 12 December 2010
Available online 20 January 2011

Keywords:
PID control
Process classification
Knowledge tuning
Performance prediction

ABSTRACT

A quadruplet, defined by the ultimate frequency ω_u, the ultimate gain k_u, the angle φ of the tangent to the Nyquist curve at the ultimate frequency and the gain $G_p(0)$, is sufficient for classification of a large class of stable processes, processes with oscillatory dynamics, integrating and unstable processes $G_p(s)$. From the model defined by the above quadruplet, a two parameter model $G_n(s_n)$ is obtained by the time and amplitude normalizations. Two parameters of $G_n(s_n)$, the normalized gain ρ and the angle φ, are coordinates of the classification ρ–φ parameter plane. Model $G_n(s_n)$ is used to obtain the desired closed-loop system performance/robustness tradeoff in the desired region of the classification plane. Tuning procedures and tuning formulae are derived guaranteeing almost the same performance/robustness tradeoff as obtained by the optimal PID controller, applied to $G_p(s)$ classified to the same region of the classification plane. Validity of the proposed method is demonstrated on a test batch consisting of stable processes, processes with oscillatory dynamics, integrating and unstable processes, including dead-time.

1. Introduction

PID controllers, in single and cascade loops, with or without derivative action, are still mostly used control systems in the majority of industrial applications. "Many strategies proposed can be easily eliminated if they are compared with well tuned PID" [1]. This is one of the reasons why a large number of tuning procedures is proposed for adjusting proportional gain k, integral gain k_i, derivative gain k_d and time constant T_f of the noise filter, in the PID controller:

$$C_{PID}(s) = \frac{k_d s^2 + ks + k_i}{s(T_f s + 1)}, \tag{1}$$

presented with transfer function $C(s)$ in Fig. 1. When k, k_i, k_d and T_f are determined, $G_{ff}(s)$ can be designed and tuned as in [2], while $C(s)$ can be implemented as in (1) or in the traditional form where noise filtering affects the derivative term only [3]. Finally, for $k_d = 0$ one obtains PI controller, with $T_f = 0$ or $T_f \neq 0$.

In practice, we are faced with the following situation: "In fact, a visit to a process plant will usually show that a large number of the PID controllers are poorly tuned" [4]. Moreover, as stated in [5]: "... 25% of all PID controller loops use default factory settings, implying that they have not been tuned at all". The reasons for the so disappointing reality are the following facts. All processes (thermo and hydro dynamic, chemical, mechanical, nuclear) in a great number of different plants, with a much greater number of operating regimes in these process plants, constitute practically an infinite batch of possible linear stable, integrating and unstable models $G_p(s)$. The complete test batch, to be used for deriving and analyzing tuning procedures for the PID controller, is practically infinite. A large test batch from [6], defining different stable and integrating processes $G_p(s)$, covers a small domain in the parameter plane proposed here for classification of processes. Accordingly, to have an adequate insight into the problem of deriving and analyzing tuning procedures for the PID controllers it is necessary to have a reliable classification of different processes $G_p(s)$.

In the present paper, firstly a simple and effective way for classification of a large class of stable, oscillatory, integrating and unstable processes, including dead-time, is developed. This is done in the proposed classification ρ–φ parameter plane, defined by the normalized gain $\rho = 1/(1+\kappa)$, $\kappa = 1/(k_u G_p(0))$ and the angle φ of the tangent to the Nyquist curve at the ultimate frequency ω_u, of a process $G_p(s)$. Recently, it has been shown in [7,8] that this additional parameter φ defines in the frequency domain an extension of the Ziegler–Nichols approach [9] to the process dynamics characterization, which makes possible to capture the essential dynamic characteristics of a large class of stable, oscillatory, integrating and unstable processes, including dead-time. It is demonstrated in [7,8] that practically the same performance/robustness tradeoff is obtained by applying the PI/PID controller constrained optimiza-

* Corresponding author. Tel.: +381 11 324 84 64; fax: +381 11 324 86 81.
E-mail addresses: tomi@etf.rs (T.B. Šekara), matausek@etf.rs (M.R. Mataušek).

0959-1524/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jprocont.2010.12.004

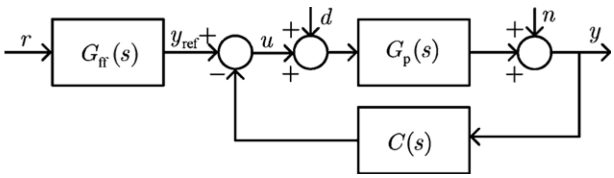

Fig. 1. Process $G_p(s)$ with the controller $C(s)$.

tion with exact $G_p(i\omega)$ or with approximation $G_m(i\omega)$, defined by the quadruplet ω_u, k_u, φ, $G_p(0)$ and model $G_m(s)$, given by

$$G_m(s) = \frac{A\omega_u e^{-\tau s}}{s^2+\omega_u^2 - A\omega_u e^{-\tau s}}\frac{1}{k_u}, \quad A = \frac{\omega_u k_u G_p(0)}{1+k_u G_p(0)}, \quad \tau = \frac{\varphi}{\omega_u}. \quad (2)$$

Secondly, PID controller tuning procedures and tuning formulae are defined for obtaining the desired performance/robustness tradeoff in the chosen region of the ρ–φ parameter plane. These procedures are based on the two parameter model $G_n(s_n)$, obtained by the time and amplitude normalizations of the model (2). Parameters k, k_i, k_d and T_f of the PID controller (1) are determined by the position of process $G_p(s)$ dynamic characteristics in the ρ–φ parameter plane. The closed-loop system performance/robustness tradeoff obtained by the proposed controller is predicted with high accuracy.

The closed-loop system time and amplitude normalization, discussed in Section 2, is necessary to activate knowledge to be used for deriving the proposed process classification and controller tuning. The classification ρ–φ parameter plane is defined in Section 3. Tuning procedures and tuning formulae, derived for the chosen regions of the ρ–φ parameter plane, are presented in Section 4. The possibility of predicting the performance/robustness tradeoff, obtained by the PID controller in the desired region of process classification, is discussed in Section 5.

2. Closed-loop system time and amplitude normalization

By approximating process $G_p(s)$ in Fig. 1 with the model (2), and by using time normalization $t_n = t\omega_u$ one obtains scaled model (2), given by

$$G_m(s_n) = \frac{1}{k_u}G_n(s_n), \quad s_n = \frac{s}{\omega_u}, \quad \rho = \frac{A}{\omega_u}, \quad (3)$$

$$G_n(s_n) = \frac{\rho e^{-\varphi s_n}}{s_n^2 + 1 - \rho e^{-\varphi s_n}}, \quad (4)$$

and the scaled PID controller $C(s)$, given by

$$C_{PID}(s_n) = \frac{k_d^* s_n^2 + k^* s_n + k_i^*}{s_n(T_f^* s_n + 1)}, \quad (5)$$

$$k^* = k, \quad k_i^* = \frac{k_i}{\omega_u}, \quad k_d^* = k_d\omega_u, \quad T_f^* = T_f\omega_u. \quad (6)$$

After time normalization, one obtains time-scaled closed-loop control system presented in Fig. 2a. By simple transformations, one obtains time and amplitude scaled closed-loop control system in Fig. 2b, where

$$C_{PID,n}(s_n) = \frac{k_{d,n}s_n^2 + k_n s_n + k_{i,n}}{s_n(T_{f,n}s_n + 1)}, \quad (7)$$

$$k_n = \frac{k^*}{k_u}, \quad k_{in} = \frac{k_i^*}{k_u}, \quad k_{dn} = \frac{k_d^*}{k_u}, \quad T_{fn} = T_f^*, \quad (8)$$

and $G_n(s_n)$ is given by (4). When parameters of the normalized PID controller (7) are determined, from (5) to (8) one obtains that parameters of the real PID controller (1) are given by

$$k = k_u k_n, \quad k_i = k_u\omega_u k_{in}, \quad k_d = \frac{k_u k_{dn}}{\omega_u}, \quad T_f = \frac{T_{fn}}{\omega_u} \quad (9)$$

An adequate tuning includes both performance issues, related to the reference and load disturbance responses. However, the reference responses can be additionally adjusted by filtering the reference, as in Fig. 1 presented by the block $G_{ff}(s)$. Moreover, this problem is efficiently solved by using the two-degrees-of-freedom implementation as in [2,8]. Because most of the PID controllers operate as regulators [1], the rejection of the load step disturbance, measured by the integrated absolute error (IAE), is of primary importance to evaluate their performance under constraints on the robustness [10]. Also, as stated in [11], derivative (noise) filter should be an integral part of PID design. The desired sensitivity to the high frequency measurement noise, closely related with T_f, is an important property of the properly tuned PID controller. Inadequate sensitivity to the high frequency measurement noise is the reason why derivative action is often excluded in the process industry control.

In the present paper PID optimization is performed under constraint on the maximum sensitivity M_s for a given value of the sensitivity to the high frequency measurement noise M_n. Parameter M_s of the of the closed-loop system in Fig. 1 is given by

$$M_s = \max_{\omega}\left|\frac{1}{1+C_{PID}(i\omega)G_p(i\omega)}\right|. \quad (10)$$

In the PID optimization $G_p(s)$ is approximated by $G_n(s_n)$, defined by the quadruplet ω_u, k_u, φ, $G_p(0)$. In this case robustness is defined by the maximum sensitivity M_{sn} given by

$$M_{sn} = \max_{\omega_n}\left|\frac{1}{1+C_{PID,n}(i\omega_n)G_n(i\omega_n)}\right|, \quad \omega_n = \frac{\omega}{\omega_u}, \quad (11)$$

which is invariant in respect to time and amplitude normalization:

$$M_s = M_{sn}. \quad (12)$$

Sensitivities to the high frequency measurement noise in the closed-loop systems in Fig. 1 (M_n) and Fig. 2b (m_n) are defined by

$$M_n = \lim_{\omega\to\infty}\left|\frac{-C_{PID}(i\omega)}{1+C_{PID}(i\omega)G_p(i\omega)}\right| = \frac{|k_d|}{T_f}, \quad (13)$$

$$m_n = \lim_{\omega_n\to\infty}\left|\frac{-C_{PID,n}(i\omega_n)}{1+C_{PID,n}(i\omega_n)G_n(i\omega_n)}\right| = \frac{k_{dn}}{T_{fn}} \quad (14)$$

From (9), (13) and (14) one obtains:

$$M_n = m_n|k_u|. \quad (15)$$

Finally, the IAE for the system in Fig. 1 and IAE_n for the system in Fig. 2b are related by

$$IAE = \frac{IAE_n}{|k_u|\omega_u}. \quad (16)$$

Taking into account the mutual relationship between closed-loop systems in Figs. 1 and 2b, presented above, for a large class of stable, oscillatory, integrating and unstable processes, analysis and optimization can be based on the analysis and optimization of the closed-loop system presented in Fig. 2b, defined by (4), (7), (12) and (14). In the present paper parameters of the normalized controller (7), k_n, k_{in}, k_{dn} and T_{fn}, are determined by using PID optimization based on the model (4), under constraints on the maximum sensitivity M_s and sensitivity to the high frequency measurement noise m_n. Then, parameters k, k_i, k_d and T_f are determined from (9).

3. Classification ρ–φ parameter plane

The proposed normalization reduces dynamics characterization of a process $G_p(s)$ to analysis of the ρ–φ model (4). From (2) and (4)