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Analytical structure and stability analysis of a fuzzy PID controller

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Abstract

Analytical structure for a fuzzy PID controller is introduced by employing two fuzzy sets for each of the three input variables and four fuzzy sets for the output variable. This structure is derived via left and right trapezoidal membership functions for inputs, trapezoidal membership functions for output, algebraic product triangular norm, bounded sum triangular co-norm, Mamdani minimum inference method, and center of sums (COS) defuzzification method. Conditions for bounded-input bounded-output (BIBO) stability are derived using the Small Gain Theorem. Finally, two numerical examples along with their simulation results are included to demonstrate the effectiveness of the simplest fuzzy PID controller.

Keywords: Fuzzy PID controllers; Analytical structure; BIBO stability

1. Introduction

Most of the industrial processes are still the conventional PID controllers due to their simple and robust design, affordable price, and effectiveness for linear systems [3,10]. So far two different configurations have been reported for PID control as shown in Fig. 1. Due to their linear structure, conventional PID controllers are usually not effective if the processes involved are higher order and time delay systems, nonlinear systems, complex and vague systems without precise mathematical models, and systems with uncertainties [3,8,10]. Till 1990 most of the research activity in fuzzy control systems area dealt with ‘‘applications’’ rather than ‘‘theory.’’ Around 1990 people began developing analytical structures for fuzzy controllers and analyzing the same, with an objective of building fuzzy control theory so that it can be applied in the similar lines of conventional control theory [21]. It has been observed that fuzzy PI [15,21] and fuzzy PD [11,14] controllers can handle the above mentioned systems better than their conventional counterparts. Fuzzy PI controllers are preferred more to fuzzy PD controllers as fuzzy PD controllers are not able to eliminate steady state errors [16]. However, fuzzy PI controllers show poor performance during the transient phase for higher order processes due to their internal integration operation. To obtain overall improved performance, fuzzy PID controllers are preferred [8,16].

A hybrid fuzzy logic proportional plus conventional integral-derivative controller in incremental form has been presented [10]. This controller is obtained by considering an incremental fuzzy logic controller in place of the proportional term in a conventional PID controller. Also a sufficient condition for BIBO stability of this controller is derived using the Small Gain Theorem. As a matter of fact, stability analysis of fuzzy control systems has been extensively discussed in [1]. This book presents several techniques for stability analysis of both Mamdani type and Takagi-Sugeno type fuzzy models. An in-depth treatment on analysis and design of fuzzy control systems via a linear matrix inequality approach has been given in [17] to deal with systems described by Takagi-Sugeno type fuzzy models.

A new fuzzy PID controller structure, based on configuration 1 [16] in Fig. 1, has been proposed. To tune the parameters of the fuzzy controller on line, a parameter adaptive method via peak observer has been presented. A two-level tuning strategy has been introduced [9] which first tries to set up the relationship between fuzzy proportional/integral/derivative gain and scaling gains at the high level, and then optimally tunes the control resolution at low level. Based on the configuration 1, a fuzzy PID controller has been suggested [7] using minimum inference engine and center average defuzzification, which behaves approximately like a parameter varying PID controller. In order to improve further the performance in

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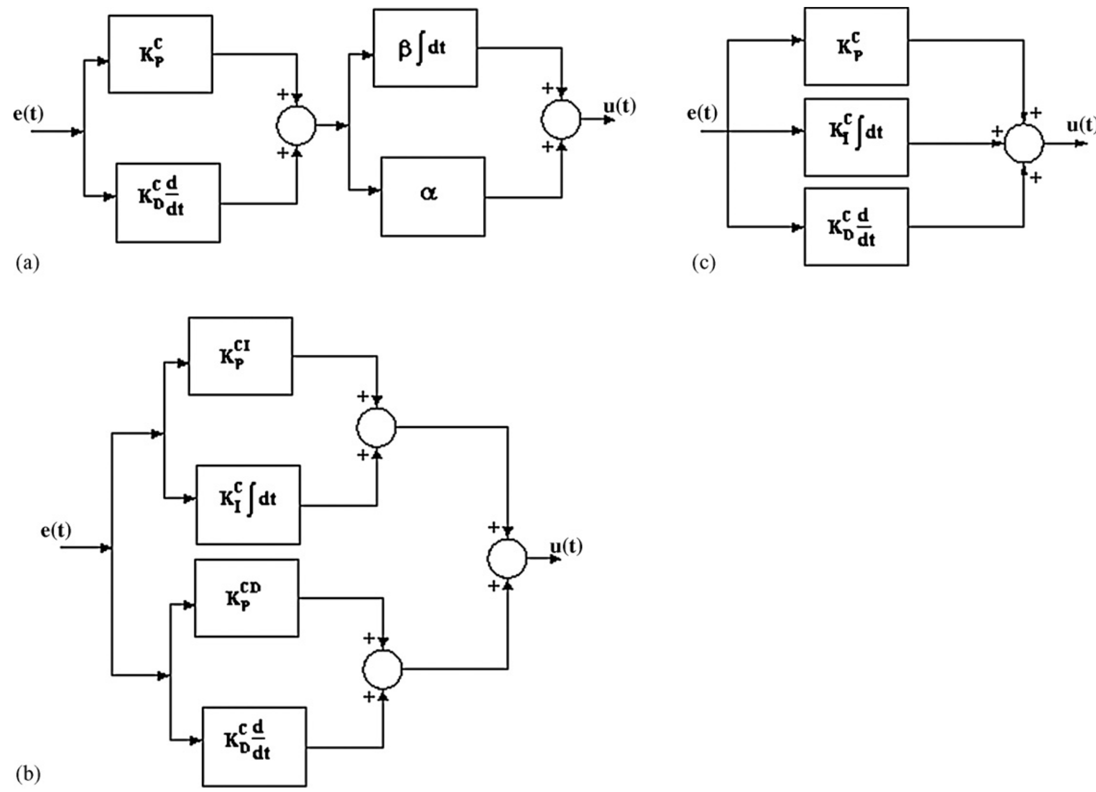

Fig. 1. PID controllers: (a) configuration 1; (b) configuration 2; (c) configuration 3.

transient and steady states, an adaptive method via function tuner has been developed [19] to tune the scaling factors of the fuzzy controller on line. An adaptive method via relative rate observer has been proposed [5] for tuning the scaling factors of the fuzzy logic controller in an on-line manner.

Fuzzy PI and fuzzy PD controllers are combined to get a fuzzy PID controller according to the configuration 2 [8] in Fig. 1. Its knowledge base consists of two-dimensional rule bases for PI and PD controls. A tuning method, based on gain margin and phase margin specifications, has been proposed [20] for determining the parameters of the fuzzy PID controller. Also sufficient conditions for BIBO stability have been determined. Several forms of decomposed proportional-integral-derivative fuzzy logic controllers (fuzzy P + fuzzy I + fuzzy D form, fuzzy PD + fuzzy I form, fuzzy PI + conventional D form, fuzzy P + conventional ID form and fuzzy PI + fuzzy PD form) have been tested and compared [4]. To obtain simple structures, the activities of the proportional, integral and derivative parts of the fuzzy PID controller are defined with simple rules in proportional rule base, integral rule base and derivative rule base.

A function-based evaluation approach has been proposed [6] for a systematic study of fuzzy PID controllers, and the fuzzy controllers have been analyzed using five simple evaluation criteria (control-action composition, input coupling, gain dependency, gain-role change, and rule/parameter growth).

It has been already proved by Mizumoto [13] that PID controllers can be obtained by using fuzzy control methods like product-sum-gravity method (algebraic product triangular norm, Larsen product inference method, bounded sum triangular co-norm and center of gravity method for defuzzification) and simplified fuzzy reasoning method (special case of product-sum-gravity method when fuzzy sets in consequent part are of same size). However, PID controller cannot be constructed by min-max-gravity method as this method gives a complicated inference result of nonlinear form for a simple fuzzy reasoning form, see Fig. 5 in [13]. It has been shown that extrapolative reasoning can also be accomplished by product-sum-gravity method and simplified fuzzy reasoning method by extending membership functions of antecedent parts of fuzzy rules [13].

In this paper attempts are made to obtain analytical structure of the fuzzy PID controller (configuration 3 in Fig. 1) by employing algebraic product triangular norm, bounded sum triangular co-norm, left (Γ-type) and right (L-type) trapezoidal membership functions for inputs, trapezoidal membership functions for output, nonlinear control rules, Mamdani minimum inference method, and COS method of defuzzification. Conditions for BIBO stability of fuzzy PID control systems are obtained. Finally, to demonstrate the superiority of fuzzy PID controller over the conventional PID controller, simulation results of two examples are included.

The paper is organized as follows. The next section deals with the fundamental components of a typical fuzzy PID controller. Section 3 presents the fuzzy PID analytical structure. Section 4 is about BIBO stability analysis of fuzzy PID control systems. Section 5 deals with design aspects of the simplest fuzzy PID controller. Section 6 includes simulation results while Section 7 consists of concluding remarks.