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Analysis on the mutual inductance of planar transformer in CMOS technology

Heng-Ming Hsu *, Chien-Wen Tseng, Hsien-Feng Liao

Department of Electrical Engineering, National Chung Hsing University, Taichung 402-27, Taiwan, ROC

Abstract

An analytical formula for calculating the mutual inductance of planar transformer has been developed in CMOS technology. The proposed formula can predict different turn ratios between primary and secondary coils. Two on-chip transformers have been measured to verify this formula, the results demonstrate the proposed formula is feasible. Furthermore, the equivalent circuit and the corresponding model are proposed to describe the high frequency behavior.
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1. Introduction

Recently, silicon-based technologies have been increasingly attractive for implementation in RF circuit applications because they are low-cost, high-yield, and rapid time-to-market. The successful implementation of all circuit blocks on a single chip depends on the key components in RF circuit [1–3]. Especially the on-chip transformers have been extensively used in high frequency operation for impedance transformation, signal coupling, DC isolation, among others [4,5].

Integrated transformers have been reported in previous efforts such as first invention by Rabjohn [6], comprehensive discussion by Long [7], silicon substrate effect by Burghartz [8], and modeling work by Mohan [9], among others [10,11]. However, all of the previous woks concentrate on characterizing the identical turn ratio between primary and secondary coils. Generally, the purpose of transformer is to transfer the voltage between two ports, the different turn ratio of transformer is frequently used in circuit design.

Therefore, we spent an effort for predicting the mutual inductance with different turn ratios in this work firstly. The investigation of mutual inductance includes both the analytical formula to predict its value in low frequency and the modeling work to describe its behavior in high frequency. An analytical formula is developed to calculate the mutual inductance of on-chip transformers. The analytical formula is developed based on the geometry mean distance (GMD) approach [12], which is a quasi-static algorithm to predict inductance in low frequency. Afterward, the equivalent circuit is proposed to depict the high frequency performance. Results of this work provide a valuable effort to design a high-performance transformer for RFIC applications.

The contents of this work are described in following section. Firstly, we make an introduction in Section 1 to describe the development history of on-chip transformer and point out the important of 1:n transformer. Secondly, the analytical formula to predict the mutual inductance of 1:1 and 1:n transformers is developed in Section 2. The advantage of analytical formula is rapid and correct to calculate the mutual inductance. It is useful during the layout design of on-chip transformer. Afterward, the model effort is paid to extract the parameters to understand the wide

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* Corresponding author. Tel.: +886 4 22854148; fax: +886 4 22851410.
E-mail address: hmhsu@nchu.edu.tw (H.-M. Hsu).

band performance of on-chip in Section 3. Finally, a brief conclusion is made in Section 4.

2. Analytical formula of mutual inductance of on-chip transformer

The concept of geometry mean distance (GMD) was introduced by Maxwell [12], then Grove extended this approach to calculate the inductance in many structures and collected these formulas to write a book in 1946 [13]. Afterward, Greenhouse implemented Grove's algorithm to planar spiral inductor for the calculation of inductance in 1974 [14]. The advantage of the GMD method is that it rapidly and accurately predicts an analytical form of the inductance. Previous studies in which the inductance of a spiral inductor is calculated by the method of GMD include [15,16]. According to the approach, the mutual inductance of planar transformer is developed firstly in this work.

In this study, due to the limited chip area, two layouts of transformer are implemented to analysis the mutual inductance. Fig. 1a depicts the layouts of on-chip transformers with identical inductance in primary and secondary coils (i.e., 1:1 case) and the corresponding die photo for RF measurement is illustrated in Fig. 1b. As described previously, the layout of different self inductances in primary and secondary coils is design in Fig. 1c for the 1:*n* case. In this layout, the primary is identical to previous configuration; however the secondary terminal is parallel with three one-turn coils. Accordingly, the self inductance is decrease due to the parallel interconnection in secondary terminal and the turn ratio will become different in this layout.

Based on the rectangular shape of the proposed transformer, the mutual inductance of transformer can be calculated by cutting each coil into horizontal and vertical strips as shown in Fig. 2. The dark and light colors in the metal strips are used to distinguish the primary and secondary coils of on-chip transformer. Each strip separates into two groups, then the resulted mutual inductance can be calculated by the GMD formula [12]. Notably the mutual inductance includes positive and negative terms due to the differently relative current direction in the metal strip. An effort is paid to derive the positive mutual inductance in following form:

$$M_{\text{positive}} = 2 \cdot \sum_{b=1}^{4N} \sum_{a=1}^{4N} (M_{\text{p}(4a-3,4b+1)} + M_{\text{p}(4a-2,4b)}), \qquad (1)$$

where the N means the total coil numbers of the individual inductor and the parameters of a and b indicate each segment related to the individual inductor.

By adopting similar manipulation for the negative term, the derived result is given by

$$M_{\text{negative}} = \sum_{b=1}^{4N} \sum_{a=1}^{4N} (M_{\text{m}(4a-1,4b-1)} + M_{\text{m}(4a-2,4b-2)} + M_{\text{m}(4a-3,4b-3)} + M_{\text{m}(4a,4b)}). \qquad (2)$$
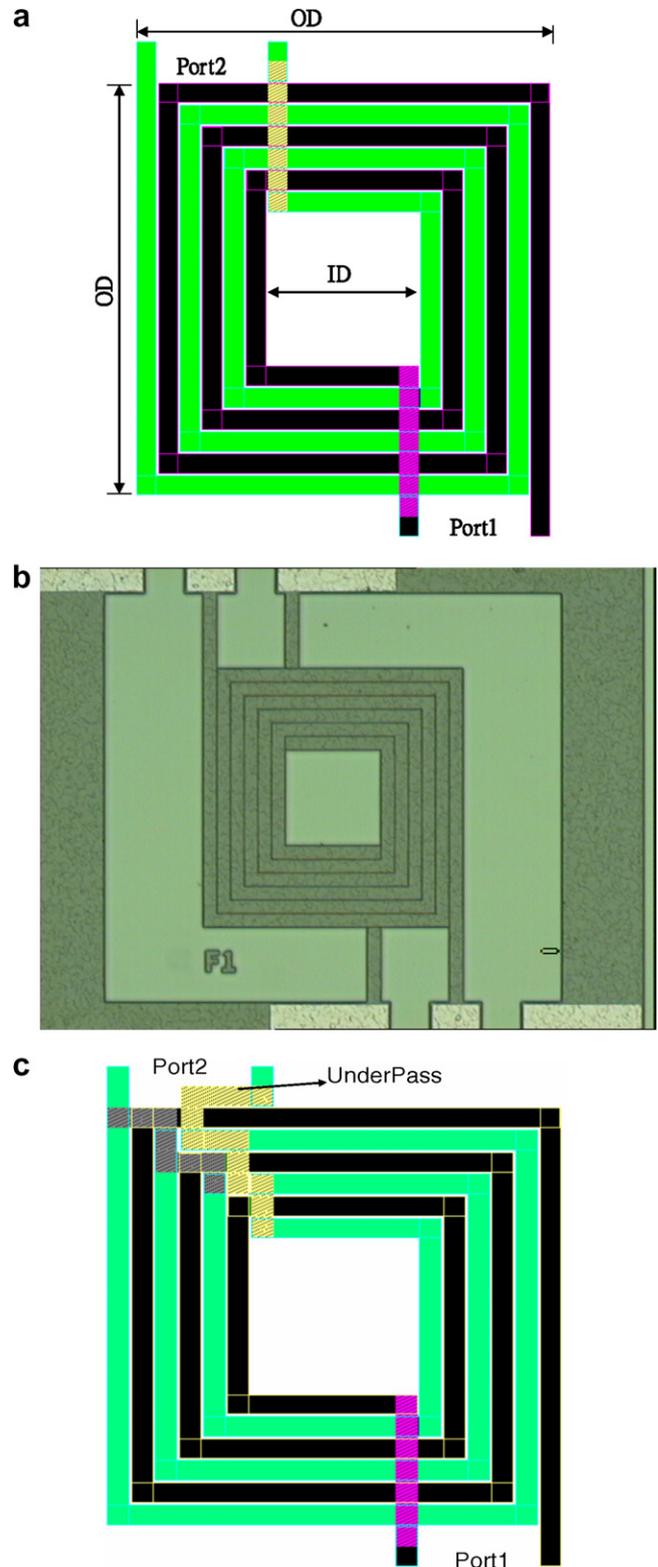

Fig. 1. Top view of proposed transformers: (a) layout of turn ratio 1:1 transformer, (b) die photo of 1:1 transformer and (c) layout of turn ratio 1:*n* transformer.

Furthermore, the formula of 1:*n* transformer is also derived in this work. Since the secondary coil is composed of parallel segments, the corresponding equivalent circuit is