Omega 38 (2010) 105–112

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

DEA-based production planning☆

Juan Du[a,*], Liang Liang[a], Yao Chen[b], Gong-bing Bi[a]

[a]School of Management, University of Science and Technology of China, 96, Jinzhai Road, Hefei, Anhui 230026, PR China
[b]College of Management, University of Massachusetts at Lowell, Lowell, MA 01845, USA

ARTICLE INFO

Article history:
Received 5 November 2008
Accepted 9 July 2009
Available online 22 July 2009

Keywords:
Production plan
Data envelopment analysis (DEA)
Decision-making units (DMUs)
Centralized decision-making environment
Efficiency analysis

ABSTRACT

Production in large organizations with a centralized decision-making environment like supermarket chains or factories with many workshops, usually involves the participation of more than one individual unit, each contributing a part of the total production. This study is motivated by a production-planning problem regularly faced by the central decision-making unit to arrange new input and output plans for all individual units in the next production season when demand changes can be forecasted. Two planning ideas have been proposed in this paper. One is optimizing the average or overall production performance of the entire organization, measured by the CCR efficiency of the average input and output levels of all units. The other is simultaneously maximizing total outputs produced and minimizing total inputs consumed by all units. According to these two ideas, we develop two DEA-based production planning approaches to find the most preferred production plans. All these individual units, considered as decision-making units (DMUs), are supposed to be able to modify their input usages and output productions. A simple numerical example and a real world data set are used to illustrate these approaches.

1. Introduction

Many researchers have modeled the production planning from various perspectives. Chazal et al. [1] study the production planning and inventory management problem based on the assumption that the firm under discussion acts in continuous time on a finite period in order to dynamically maximize its instantaneous profit. By assuming that the storage cost is convex, they have related the optimal planning problem to the study of a backward integro-differential equation, from which an explicit construction of the optimal plan can be obtained. Using mixed-integer linear programming models, Pastor et al. [2] model a production planning problem for a wood-turning company in order to meet the demand at minimum cost while subjected to a series of principal conditions.

This paper attempts to look at the production planning problem from the productivity and efficiency perspective using data envelopment analysis (DEA). Developed by Charnes et al. [3], DEA is an effective approach for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and outputs. In recent years, DEA has been applied to DMUs in many different settings, such as efficiency and effectiveness in railway performance [4], R&D project evaluation [5], productivity evaluation on research institutions [6], hotel chain performance [7], and even evaluation for Olympic achievements [8]. For more DEA application examples, see Handbook on Data Envelopment Analysis, edited by Cooper, Seiford and Zhu [9].

Using DEA in production planning or alike studies is relative new. Golany [10] presents an interactive multi-objective linear programming procedure to generate a set of alternative efficient points for DMU to consider. The procedure depends on empirical production functions generated by DEA and then adjusted by new information provided by the decision-maker in each iteration. The new solution (DMU) then helps decision-makers in setting targets for desired outputs. Beasley [11] presents non-linear resource-allocation models to jointly decide input and output amounts for each DMU for the next period while maximizing the average efficiency of all DMUs. Korhonen and Syrjanen [12] develop an interactive approach based on DEA and multiple-objective linear programming technique to a resource-allocation problem that typically appears in a centralized decision-making environment. By simultaneously maximizing the total amount of output variables produced by each individual unit, an efficient resource-allocation solution as well as all the output amounts can be obtained.

Similar to the work of Korhonen and Syrjanen [12], the current paper also considers a centralized decision-making environment in the sense that in a large organization there are a set of n homogenous individual units operating under a central unit which acts as a supervisor. All these individual units produce same set of outputs at the cost of same set of inputs. The central unit makes production

☆Processed by Associate Editor Huang.

* Corresponding author. Tel.: +86 13866111161.
E-mail addresses: merrydj@mail.ustc.edu.cn (J. Du), lliang@ustc.edu.cn (L. Liang), yao_chen@uml.edu (Y. Chen), bgbwhl@ustc.edu.cn (G.-b. Bi).

0305-0483/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.omega.2009.07.001

plans for all the individual units based upon the forecasted demands for the future.

With the forecasted demands for the outputs, the paper develops DEA-based models to determine new production plans for all the individual units under a centralized decision-maker. For example, a bank's cooperate management decides a sales plan for its branches with respect to the number of credit cards to be issues and number of loans to be processed. Corresponding to two different planning ideas, the central unit determines a new production plan in such two ways that (1) the average or overall production performance in the entire organization will be optimized after planning, which means maximizing the efficiency of the average input and output levels of all individual units under the framework of DEA, and (2) within the original production possibility set, the total outputs produced by all units will be maximized, and at the same time the total inputs consumed will be minimized. For Idea 1, we consider three demand change cases and their combinations. First, we assume that all demand changes for all outputs are either positive or negative. We then relax this requirement and assume that some demand changes are positive, some are negative and the rest are zero, i.e., no demand change is predicated. As for Idea 2, these three demand change situations can be incorporated into one model.

The rest of this paper is organized as follows. Section 2 takes the perspective of Idea 1, and develops a planning approach based on CCR efficiency analysis and several production planning models fitting different situations in demand changes. Section 3 proposes a DEA-based production planning model from Idea 2, and proves an important property concerning the CCR efficiency of the new plans. Both approaches in Sections 2 and 3 are demonstrated by a same simple numerical example. Section 4 applies the two approaches to real world data set consisting of 20 fast food restaurants. Section 5 concludes.

2. Planning models for optimizing overall performance—Idea 1

Before we develop our DEA-based production planning models, we assume that there are a set of n DMUs and unit j is denoted by DMU_j $(j=1,\dots,n)$. The ith input and rth output of DMU_j $(j=1,\dots,n)$ are denoted by x_{ij} $(i=1,\dots,m)$ and y_{rj} $(r=1,\dots,s)$, respectively. The original CCR efficiency of all DMUs can be evaluated by the following DEA model (1) [3], which can be transformed into a linear program.

$$\begin{cases} \max & h_j=\dfrac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}} \\ \text{s.t.} & \dfrac{\sum_{r=1}^{s}u_r y_{rk}}{\sum_{i=1}^{m}v_i x_{ik}} \le 1, \quad k=1,\dots,n \\ & v_i \ge \varepsilon, \quad i=1,\dots,m \\ & u_r \ge \varepsilon, \quad r=1,\dots,s \end{cases} \tag{1}$$

Next we assume that the upper demand change for output r $(r=1,\dots,s)$ in the next production season can be forecasted as $\tilde{D}_r$. Here in this paper, all $\tilde{D}_r$, $r=1,\dots,s$ can be either positive or negative, corresponding to an increase or a decrease in the demand for output r. To meet the demand changes, the central unit will determine the most preferred input–output plans for all DMUs. The current study in this section chooses to maximize the CCR efficiency of average input and output levels of all DMUs. The reason for choosing this planning principle is that maximizing the CCR efficiency of the average input and output levels of all DMUs ensures the average or overall production capability in the entire organization to exert its highest potential after planning.

We note that demand change for each output can be different. We can have (1) all demand changes are positive or negative, (2) some demand changes are positive, some are negative, and the rest can be zero. Next we will develop several production planning models reflecting the above different demand change situations.

2.1. Models for all positive or negative demand changes

Assume that demand changes for all outputs $\tilde{D}_r$, $r=1,\dots,s$ in the next production season are in the same direction, or more specifically, $\tilde{D}_r$, $r=1,\dots,s$ are all either positive or negative. Since constant returns to scale (CRS) is assumed in model (1), we assume that all inputs and outputs of DMU_j will change by the same proportion denoted by $\Delta_j \ge -1$ $(j=1,\dots,n)$. If $\tilde{D}_r \ge 0$, then $\Delta_j \ge 0$, and if $\tilde{D}_r \le 0$, then $-1 \le \Delta_j \le 0$. Further, we have

$$\sum_{j=1}^{n}\Delta_j y_{rj} \le \tilde{D}_r \quad (r=1,\dots,s)$$

and $((1+\Delta_j))x_{1j},\dots,(1+\Delta_j)x_{mj};\ (1+\Delta_j)y_{1j},\dots,(1+\Delta_j)y_{sj})$ as the new production plan forDMU_j. Here $(1+\Delta_j)$ can all be either greater than or less than one, corresponding to either an increased or decreased production size.

Based upon model (1) and the average input and output levels of all DMUs, we have the following planning model (2) when all demand changes occur in the same direction.

$$\begin{cases} \max & \theta=\dfrac{\sum_{r=1}^{s}u_r\left[\frac{1}{n}\sum_{j=1}^{n}(1+\Delta_j)y_{rj}\right]}{\sum_{i=1}^{m}v_i\left[\frac{1}{n}\sum_{j=1}^{n}(1+\Delta_j)x_{ij}\right]} \\ \text{s.t.} & \dfrac{\sum_{r=1}^{s}u_r\left[\frac{1}{n}\sum_{j=1}^{n}(1+\Delta_j)y_{rj}\right]}{\sum_{i=1}^{m}v_i\left[\frac{1}{n}\sum_{j=1}^{n}(1+\Delta_j)x_{ij}\right]} \le 1 \\ & \dfrac{\sum_{r=1}^{s}u_r(1+\Delta_j)y_{rj}}{\sum_{i=1}^{m}v_i(1+\Delta_j)x_{ij}} \le 1, \quad j=1,\dots,n \\ & \sum_{j=1}^{n}\Delta_j y_{rj} \le \tilde{D}_r, \quad r=1,\dots,s \\ & v_i,u_r \ge \varepsilon, \quad i=1,\dots,m,\ r=1,\dots,s \\ & \begin{cases} \Delta_j \ge 0 & \text{when } \tilde{D}_r \ge 0, \\ -1 \le \Delta_j \le 0 & \text{when } \tilde{D}_r \le 0, \end{cases} \quad r=1,\dots,s,\ j=1,\dots,n \end{cases} \tag{2}$$

Model (2) is equivalent to the following model:

$$\begin{cases} \max & \theta=\dfrac{\sum_{r=1}^{s}u_r\sum_{j=1}^{n}(1+\Delta_j)y_{rj}}{\sum_{i=1}^{m}v_i\sum_{j=1}^{n}(1+\Delta_j)x_{ij}} \\ \text{s.t.} & \dfrac{\sum_{r=1}^{s}u_r\sum_{j=1}^{n}(1+\Delta_j)y_{rj}}{\sum_{i=1}^{m}v_i\sum_{j=1}^{n}(1+\Delta_j)x_{ij}} \le 1 \\ & \dfrac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}} \le 1, \quad j=1,\dots,n \\ & \sum_{j=1}^{n}\Delta_j y_{rj} \le \tilde{D}_r, \quad r=1,\dots,s \\ & v_i,u_r \ge \varepsilon, \quad i=1,\dots,m,\ r=1,\dots,s \\ & \begin{cases} \Delta_j \ge 0 & \text{when } \tilde{D}_r \ge 0, \\ -1 \le \Delta_j \le 0 & \text{when } \tilde{D}_r \le 0, \end{cases} \quad r=1,\dots,s,\ j=1,\dots,n \end{cases} \tag{3}$$

By comparing models (2) and (3), we note that the CCR efficiency of the average input and output levels of all DMUs is equivalent to the efficiency of the total input consumption and output production.

After making the change of variables $\mu_r=tu_r$ and $\omega_i=tv_i$, where

$$t=\left[\sum_{i=1}^{m}v_i\sum_{j=1}^{n}(1+\Delta_j)x_{ij}\right]^{-1},$$