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PCPDIPM based optimal reactive power flow model with discrete variables

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ABSTRACT

This paper presents a new approach to deal with the optimal reactive power flow (ORPF) problem with the discrete control variables. First, a quadratic ORPF model based on augmented rectangular coordinates is established by treatment with the TLC branch; and then quadratic penalty functions are incorporated into the proposed model to handle the discrete control variables; at last, the predictor corrector primal dual interior point method (PCPDIPM) is used to implement the optimization.

In the PCPDIPM based ORPF solution, the quadratic discretization formulation results in the constant Hessians that all have elements of 1, or −1, or the penalty factor, and mostly being zero, thereby accelerating the entire optimal process significantly. Experimental results are provided comparing the performance of the proposed discretization approach with that of the conventional one.

Introduction

The main purpose of optimal reactive power flow is to minimize the total power losses of the network by scheduling available control parameters, without violating a set of nonlinear equality and inequality constraints. It is complicated nonlinear problem in realistic applications by the presence of a large number of discrete variables [1–4]. Considering its significance in power systems planning and operation practice [5], many researchers have paid intensively attention to the discretization optimization.

In the past three decades, various algorithms were developed for the optimization, e.g., round-off techniques [6,7], penalty methods [8,9], and heuristic algorithms [10–13]. In the simplest round-off technique [6,7], the underlying idea is to firstly solve the OPF by treating all variables as continuous, and then the discrete variables are rounded off to the nearest discrete values. At last, freeze the discrete variables and determine the remaining continuous variables by solving the conventional power-flow problem. Owing to blind action, the round-off approach may lead to an infeasible OPF problem for the continuous variables, or poor sub-optimal solutions of the OPF problem. A penalty method based on polar coordinates is presented in [9], in which penalty functions are incorporated into the PCPDIPM to realize successive discretization of the discrete control variables. Although the approach is robustness, the Hessians associated with constraints, updated in each iteration, lead to time-consuming, especially for the large-scale systems. Last but not least, the reference [10] proposes a heuristic approach to cope with discrete variables with two procedures. The approach firstly sets the discrete variables according to the sensitivities of the objective and inequality constraints with respect to discrete variables, and then it uses an OPF to re-optimize the continuous variables only. However, inadequate discrete variable settings may also lead to an infeasible OPF problem for the continuous variables. Other heuristic algorithms [11–13], such as Tabu search, genetic algorithm and ordinal optimization theory based algorithm, are global optimization methods but are slower convergence and poor robustness.

Recently, the predictor corrector primal dual interior point method (PCPDIPM) [14–21], due to its robustness and fast computation, has been applied widely to compute the optimization problems in large-scale power systems. Based on PCPDIPM in itself, many researchers propose some techniques to speed up the calculation, such as rearranging the correction equation and optimizing node ordering. However, for higher-than-two-order nonlinear problem, such as the ORPF based on polar coordinate, the PCPDIP method need to update the Hessians associated with the objective function and constraint conditions in each iteration, which requires a certain computation burden. On the other hand, ORPF solution approaches treat conventionally all of the variables as

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Notations

$\dot{V}_i = e_i + jf_i$ complex voltage at bus i
$\dot{I}_i = a_i + jb_i$ originally complex current injected at bus i
$Y_{ij} = G_{ij} + jB_{ij}$ ijth element of the nodal admittance matrix
$y_t = g_t + jb_t$ LTC branch admittance
$P_{Gi} + jQ_{Gi}$ complex power output of the generator at bus i
$P_{Li} + jQ_{Li}$ complex power demand at bus i
Q_{Ci} reactive power output of the compensator at bus i
N set of original buses
C set of all artificial buses, normal side buses, and off normal side buses
G set of all generators
F set of all shunt elements
T set of LTC branches with a two-winding
t transformer turn ratio;
n number of all original buses;
n_t number of LTC branches with a two-winding
g number of all generators
r number of all shunt elements
p number of all discrete variables

continuous ones, but there are discrete control variables in power systems. So how to propose a discrete ORPF model suitable for PCPDIPM has as well been a meaningful work.

It is well known that the ORPF formulation based on rectangular coordinate could be quadratic form [5]. The advantages of a quadratic function are [14–16]: (a) its Hessian is constant, (b) its Taylor expansion terminates at the second-order term without truncation error, and (c) the higher-order terms are easy to approximate. The quadratic formulation using rectangular coordinates allows for ease of matrix setup and higher order information in PCPDIPM process that helps improve IPM performance. Due to the existence of tap ratio in LTC branch, the optimal power flow model in [14] is not fully quadratic. A completely quadratic model of ORPF is presented in [16]. In that paper, the authors employ a dummy node located between the ideal transformer and its series impedance to build the ORPF model in rectangular coordinates.

In this paper, we present a quadratic ORPF model based on augmented rectangular coordinates for the ORPF problem including discrete control variables. In the PCPDIPM based ORPF solution, the quadratic formulation results in the constant Hessians that need to be calculated only once in the entire optimal process, thereby accelerating the entire optimal process significantly. In short, the underlying ideas are as follows:

- First, the load tap changing (LTC) transformer is represented by an ideal transformer and its series impedance with a fictitious node located between them. The artificial node voltage and the bus current injections at the LTC two end nodes are used to describe the LTC branch equations, and the constraint equations relating the current injections and voltages between two sides, are formed.
- Next, the nodal voltage equations and the bus power constraints are retained.
- At last, quadratic penalty functions are incorporated into the proposed model to handle the discrete control variables.

The paper is consists of five sections. Section 'Mathematical model of ORPF' introduces the novel rectangular ORPF formulation with discrete control variables. Section 'PCPDIPM algorithm for ORPF' gives outline of the PCPDIPM algorithms which is used with the novel formulation. Section '**Simulation results**' displays numerical performances achieved by the approach. At the end, conclusions are made in section '**Conclusion**'

Mathematical model of ORPF

For the LTC branch, the classical model is shown in Fig. 1, where t is the transformer turns ratio, and y_t is the branch admittance. The branch equations can be written in rectangular form as follows [15]:

$$a_{Tij} = g_t\left(e_i - \frac{1}{t}e_j\right) + b_t\left(-f_i + \frac{1}{t}f_j\right) \tag{1}$$

$$b_{Tij} = g_t\left(f_i - \frac{1}{t}f_j\right) + b_t\left(e_i - \frac{1}{t}e_j\right) \tag{2}$$

$$a_{Tji} = g_t\left(-\frac{1}{t}e_i + \frac{1}{t^2}e_j\right) + b_t\left(\frac{1}{t}f_i - \frac{1}{t^2}f_j\right) \tag{3}$$

$$b_{Tji} = g_t\left(-\frac{1}{t}f_i + \frac{1}{t^2}f_j\right) + b_t\left(-\frac{1}{t}e_i + \frac{1}{t^2}e_j\right) \tag{4}$$

It can be seen the above equations are higher-than-two-order because of the tap settings t. This seems to eliminate the advantage of the rectangular-based ORPF, which can be formulated in quadratic functions [15].

In the paper, an artificial node m is introduced in between the ideal transformer and the series impedance. The artificial node voltage $\dot{V}_m = e_m + jf_m$, the bus current injections $\dot{I}_i = a_i + jb_i$ and $\dot{I}_j = a_j + jb_j$, at the LTC two end nodes i and j are used to describe the LTC branch equations, as shown in Fig. 1. Meanwhile, the artificial node voltage and bus injection currents are used as one of the optimization variables. The LTC branch equations between nodes i and m can be written as:

$$-b_tf_m + g_te_m + b_tf_i - g_te_i - a_i = 0 \quad i, m \in C \tag{5}$$

$$g_tf_m + b_te_m - g_tf_i - b_te_i - b_i = 0 \quad i, m \in C \tag{6}$$

With lossless ideal transformer, we have:

$$f_j - t_{jm}f_m = 0 \quad j, m \in C \tag{7}$$

$$e_j - t_{jm}e_m = 0 \quad j, m \in C \tag{8}$$

$$b_m + t_{jm}b_j = 0 \quad j, m \in C \tag{9}$$

$$a_m + t_{jm}a_j = 0 \quad j, m \in C \tag{10}$$

Eqs. (5) and (6) are the branch constraints. Eqs. (7) and (8) are the voltage constraints, relating the voltages at both sides; Eqs. (9) and (10) are the current constraint conditions on the currents in both sides. All these six equations are in quadratic form.

Thus, for the power system with n original buses, the nodal voltage equations at the i-th bus can be written as:

$$\sum_{j\in i}(-B_{ij}f_j + G_{ij}e_j) - \sum_{k\in i}a_{Tk} - a_i = 0 \quad i \in N \tag{11}$$

$$\sum_{j\in i}(G_{ij}f_j + B_{ij}e_j) - \sum_{k\in i}b_{Tk} - b_i = 0 \quad i \in N \tag{12}$$

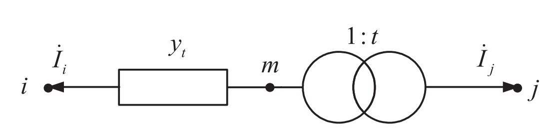

Fig. 1. Model of the LTC branch.