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Allocation of reactive power support based on a voltage source model

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ABSTRACT

This paper presents a method for the allocation of reactive power support based on the electric circuit theory. By assuming that a generator is a voltage source and applying the superposition theorem, it is possible to identify the reactive power contributions made by each source in order to meet the load demand of each load bus. Models based on voltage sources have the property of reproducing the local nature of the reactive power/voltage and lead to coherent and easily interpretable allocations. In addition, the expression for the allocation of costs avoids the presence of cross-subsidies. The method was validated with the use of two test-systems and the results were compared with those of another method.

1. Introduction

As a result of the economic decentralization of electric power systems worldwide, all the participants in the electricity market have open access to the transmission network [1–3]. In this scenario, the ancillary services associated with the electric power system operation became more important in view of the role they played in ensuring that systems would continue to function properly. However, since the costs of these services must be distributed among those who use them, the fairness of this distribution has become a highly relevant issue.

As an ancillary service, the role of reactive power support is to maintain the proper voltage profile [1], and a fair allocation of the costs of the providers of this service offers a number of advantages. For example, it helps the participants in the market to make efficient investments in reactive power sources so as to ensure a safer operation.

Because of the great analytical complexity of the problem several cost allocation strategies are proposed. A consensus has not yet been reached and one would be the most welcomed. In [4], allocation is considered according to game theory. In [5–8] the proposal is to allocate costs by electricity flow tracing, in which case, based on the principle of proportional division, approximations are deduced for calculating the active or reactive power that a generator supplies to the system loads. In [9], a method for tracing the output current as well as the output power of individual generators in the power grid is proposed. The method is based on a principle of proportional sharing of complex power, which is proven to be in conformity with the circuit theory.

In [10,11], the proposal is to determine the contributions from the sources to the power flows by simplifying the system into state graphs. Recursive equations are then derived and they are used to calculate the active and reactive power portions that each source contributes to the loads. In [12], based on ac load flow solution, a method is suggested which can trace downstream and upstream power flow paths and calculate the contribution factors of generations and loads to the line flows. The power transfer between generators and loads can also be determined. In [13], by translating all power injections into real and imaginary currents, the proposed method traces them to determine how much current each source supplies to each sink. The current contributions are then translated into contributions to the active and reactive power output of the generators. The decomposition is reasonably accurate for the reactive power generation. In [14], a nodal generation distribution factors method is used by applying a search algorithm for determining the share of every generator in the particular line power flow. The proposal in [15] is an interesting method based on the circuit theory and on the partitioning of the Y matrix in order to decompose the voltage in the load buses and calculate the allocation of reactive power. An expression is then deduced, where the voltage of each load bus is a linear combination of the generator voltages. However, in this case, the allocation proposed not only gives rise to cross-subsidies, but also allocates responsibility to loads with no reactive power demand.

The method presented in this paper solves such problems because it avoids cross-subsidies and assigns responsibilities that are consistent with the demand. The proposed method – called *AMvVS* (Allocation Method via Voltage Source) – calculates the

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reactive power support allocation based on circuit laws and explores a property that makes it possible to deduce the proposed allocation in a simple and direct manner. The method is validated and compared with [15] with the use of a 5-bus system and then it is applied to the 30-bus system.

The main attributes of the proposed method, which are:

(a) allocates reactive power support responsibilities;
(b) because it is based on the voltage source concept, it reproduces the local nature of reactive power in a suitable manner;
(c) it is simple and easy to understand because it is based on circuit laws;
(d) cost allocation does not contain cross-subsidies;
(e) the allocation obtained is coherent with electrical distance.

2. Description of the method

Let a system have N buses, where NL are load buses and NG are generation buses, and a known operation point, for example, resulting from the calculation of a load-flow problem. The branches of the system are represented as Fig. 1. The nodal equations for current injections may be expressed in a partitioned way by:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \cdot \begin{bmatrix} E_G \\ E_L \end{bmatrix} \tag{1}$$

where I_G, I_L, E_G and E_L are the current and voltage vectors of the generation and load buses, respectively.

Considering the loads as constant admittances, the bottom part of (1) may be rewritten as follows:

$$0 = Y_{LG}E_G + Y'_{LL}E_L \tag{2}$$

where Y_{LG}: admittance matrix considering the connections of the load buses to the generator buses; Y_{LL}: admittance matrix considering the connections among the load buses; Y_{GG}: admittance matrix considering the connections among the generation buses; E_G, E_L: nodal voltage vectors of the generation and load buses, respectively, as defined above; Y'_{LL} is the admittance matrix among load buses after the load equivalent admittances have been added to the diagonal elements.

The load bus voltages are calculated from (2) according to the generator voltages:

$$E_L = [Y'_{LL}]^{-1} Y_{LG} \cdot E_G = D \cdot E_G \tag{3}$$

where $D = [Y'_{LL}]^{-1} Y_{LG}$.

According to the superposition principle, the voltage $E_{k,}$ in a load bus k, may be represented as the following sum:

$$E_k = \sum_{b=1}^{NG} D_{k,b} \cdot E_G^b \tag{4}$$

and it may be assumed that:

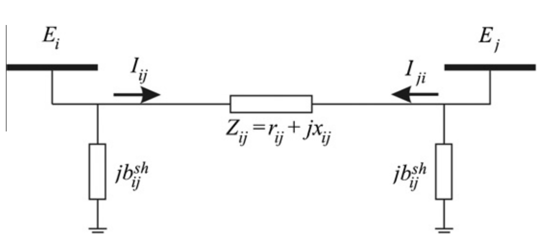

Fig. 1. Representation of the transmission branch.

$$E_k^b = D_{k,b} \cdot E_G^b \tag{5}$$

where E_k^b is the voltage contribution of load bus k resulting from the action of source b only. E_G^b is the voltage in the generating bus b and $D_{k,b}$ is the element of the kth line and bth column of the [D] matrix.

It may be observed that the superposition principle has been satisfied:

$$E_k = \sum_{b=1}^{NG} E_k^b, k = 1, \ldots, NL \tag{6}$$

2.1. Allocation of reactive power support costs

For a load bus k, $k = 1, \ldots, NL$, the complex power demand may be given by:

$$S_k = E_k I_k^* \tag{7}$$

where S_k and I_k are the complex power demand and complex current of bus k respectively.

Eq. (7) can also be rewritten as:

$$S_k = E_k (E_k Y_k)^* = Y_k^* E_k (E_k)^* \tag{8}$$

substituting (6) in (8) one has:

$$S_k = Y_k^* \left[\left(\sum_{b=1}^{NG} E_k^b \right) \left(\sum_{b=1}^{NG} (E_k^b)^* \right) \right] \tag{9}$$

Eq. (9) may be rewritten as:

$$S_k = Y_k^* \left\{ \sum_{b=1}^{NG} \left[E_k^b (E_k^b)^* + E_k^b \sum_{m=1;m\neq b}^{NG} (E_k^m)^* \right] \right\} \tag{10}$$

or:

$$S_k = Y_k^* \left[\sum_{b=1}^{NG} |E_k^b|^2 + \sum_{b=1}^{NG} E_k^b \sum_{m=1;m\neq b}^{NG} (E_k^m)^* \right] \tag{11}$$

By separating the voltages into real and imaginary parts:

$$S_k = Y_k^* \left[\sum_{b=1}^{NG} |E_k^b|^2 + \sum_{b=1}^{NG} |E_k^b| \sum_{m=1;m\neq b}^{NG} |E_k^m| \cos(\theta_k^{bm}) + j \sum_{b=1}^{NG} |E_k^b| \sum_{m=1;m\neq b}^{NG} |E_k^m| \sin(\theta_k^{bm}) \right] \tag{12}$$

where $\theta_k^{bm} = \theta_k^b - \theta_k^m$ represents the angular difference of the phasors E_k^b and E_k^m.

It may be proven (see Appendix A) that:

$$\sum_{b=1}^{NG} |E_k^b| \sum_{m=1;m\neq b}^{NG} |E_k^m| \sin(\theta_k^{bm}) = 0 \tag{13}$$

Expression (12) may be rewritten as:

$$S_k = Y_k^* \left[\sum_{b=1}^{NG} |E_k^b|^2 + \sum_{b=1}^{NG} |E_k^b| \sum_{m=1;m\neq b}^{NG} |E_k^m| \cos(\theta_k^{bm}) \right] \tag{14}$$

It may be noticed that the term within the square brackets is a real number. Therefore, the complex nature of S_k depends on Y_k.

This expression represents the contributions of the NG sources to meeting the demand of the load bus k.

When individualized, the contribution of the bth source to load k is given by:

$$S_k^b = Y_k^* \left[|E_k^b|^2 + |E_k^b| \sum_{m=1;m\neq b}^{NG} |E_k^m| \cos(\theta_k^{bm}) \right] \tag{15}$$

Considering that $Y_k = G_k + jB_k$, the reactive power demand allocation of the bth source for bus k is calculated by: