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A new computational method for reactive power market clearing

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ABSTRACT

After deregulation of electricity markets, ancillary services such as reactive power supply are priced separately. However, unlike real power supply, procedures for costing and pricing reactive power supply are still evolving and spot markets for reactive power do not exist as of now. Further, traditional formulations proposed for clearing reactive power markets use a non-linear mixed integer programming formulation that are difficult to solve.

This paper proposes a new reactive power supply market clearing scheme. Novelty of this formulation lies in the pricing scheme that rewards transformers for tap shifting while participating in this market.

The proposed model is a non-linear mixed integer challenge. A significant portion of the manuscript is devoted towards the development of a new successive mixed integer linear programming (MILP) technique to solve this formulation. The successive MILP method is computationally robust and fast. The IEEE 6-bus and 300-bus systems are used to test the proposed method. These tests serve to demonstrate computational speed and rigor of the proposed method.

1. Introduction

North American power systems have been restructured over the past decade to allow competition. In doing so, the vertically integrated structure has been functionally resolved into several independent corporations for generation (GenCo), transmission (TransCo) and distribution (DisCo). The US Federal Energy Regulatory Commission (FERC) identifies six essential components of electricity supply as ancillary services that are essential to complete a real power trade [1]. The independent system operator (ISO) contracts to provide these services through control signals and receives them through P/Q flow. The ancillary services, which are intrinsically entwined with real power, have to be separately priced and charged according to their consumption. However, in most of the restructured power systems, independent mechanisms to trade in all the ancillary services are yet to be fully developed. They are still priced in terms of the associated real power and not based entirely on their own merit. This in turn results in a benefit for some and a loss for others, preventing fair trade. Hence, unbundling of ancillary services and forming independent markets for each service is a pressing priority.

Reactive power supply is an ancillary service that has to be planned for the future and priced for trade in an electricity market. A recent FERC report [2] has initiated discussion on the task of reactive power supply pricing and planning, emphasizing its complexity and urgent need. Power system optimization algorithms are the key tools for pricing of reactive power supply. These algorithms must undergo a paradigm shift in their definition from the former vertically integrated version to a new model to address the challenges of the restructured market.

1.1. Literature survey

Reactive power by itself does not consume energy; however, its generation and transfer cause real power loss and hence consume energy. Further, it occupies partial or full operational capacity of equipment through which it traverses. As it occupies equipment capacities and causes energy loss, it is associated with capital and operating costs. In a transmission system with high X/R ratio, reactive power flows largely on the command of voltage gradient and thus it is intrinsically entwined with voltage profile, stability and security.

As restructuring of electricity markets is a recent occurrence, a few works that model reactive power markets [3–6] have been reported. Ahmed and Strbac [6] present an annualized cost model. Zhong et al. [7–9] have presented three works modeling grid-wide and localized reactive power markets incorporating a framework to handle generators' *lost opportunity costs* and transmission losses. Reactive power scheduling remains an active research area [12–14].

The FERC document [2] envisages a complete reactive power market clearing scheme addressing issues such as (a) capital investment, (b) operating costs (losses) and (c) voltage security. The method must be computationally efficient.

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Nomenclature

NG number of generators
NLB number of load buses
NS number of var sources
NTR the number of transformers
KG_{ik} and KGo_{ik} cost curve coefficients of the ith generator in the kth state
KS_i and KSo_i cost curve coefficients of the ith var source
KTo_i cost of moving tap setting of the ith transformer
KL cost of real power in $/MW
P_i real power injection into the ith bus given by the sum of powers flowing in the connected lines
PL the system transmission loss
QGC_i cost of the reactive power supply from the ith generator
QG vector of generator reactive power output
QGo vector of present state of generator reactive power output
QSC_i cost of the reactive power from the ith var source
QS vector of reactive power output from var sources
QSo vector of present state of reactive power output from var sources
T vector of tap setting of transformers
To vector of base case value of tap position of transformers
Tx vector of tap position of transformers before the start of an MILP's in the successive MILPs
UG_{ik} status of the ith generator's operational state $\in\{0,1\}$
US_i status of the ith var source $\in\{0,1\}$
UT_i denotes a tap shift in the ith transformer $\in\{0,1\}$
UGo_{ik} present state status of the ith generator's operational segment $\in\{0,1\}$
USo_i present state status of the ith var source $\in\{0,1\}$
VG vector of generator bus voltage magnitude
VGo vector of generator bus voltage magnitude in its present state
VL a vector of load bus voltage magnitude
X $[VG^t QS^t T^t]^t$
Y $[QG^t VL^t]^t$
δ vector of bus voltage phase angles
ΔQG_{ik} change in reactive power output of the ith generator when it operates in the kth segment
ΔCG, ΔCS, ΔCT, ΔPL, ΔVG, ΔQS, ΔT, ΔQG, ΔVL, ΔX, ΔY and $\Delta\delta$ changes in CG, CS, CT, PL, VG, QS, T, QG, VL, X, Y and δ, respectively
$\underline{VG}_i\,(\overline{VG}_i)$, $\underline{QS}_i\,(\overline{QS}_i)$, $\underline{T}_i\,(\overline{T}_i)$, $\underline{QG}_{ik}\,(\overline{QG}_{ik})$, $\underline{VL}_i\,(\overline{VL}_i)$, $\underline{\Delta VG}_i\,(\overline{\Delta VG}_i)$, $\underline{\Delta QS}_i\,(\overline{\Delta QS}_i)$, $\underline{\Delta T}_i\,(\overline{\Delta T}_i)$ and $\underline{\Delta VL}_i\,(\overline{\Delta VL}_i)$ min (max) limits on VG_i, QS_i, T_i, QG_{ik}, VL_i, ΔVG_i, ΔQS_i, ΔT_i and ΔVL_i, respectively
$\overline{\underline{VG}}$, $\overline{\underline{QS}}$ and $\overline{\underline{T}}$ step size restrictions on changes in ΔVG, ΔQS and ΔT, respectively in a LPMOVE
$[D]$ a linear matrix relation between ΔVL and ΔX
$[E]$ a linear matrix relation between ΔQG and ΔX
CL a linear matrix relation between ΔX and ΔPL
PD and QD are vectors of real and reactive power loads at buses
CG, CS and CT are cost functions for reactive power supply or related service from generators, reactive power sources and transformers
F() represents a set of power balance equations at all the buses in the power system
Subscript 'o' on any variable in general refers to the state of the variable before optimization

1.2. Objective of this work

While all of the methods surveyed above settle markets in one way or the other and remunerate generators and capacitors (a part of transmission system), they do not reward transformers for their role in the reactive power market operations. Thus, inclusion of transformer as a resource and development of an appropriate mechanism to reward them is an objective of this work. Further, these referred settlement schemes are non-linear mixed integer programming (NMIP) formulations [7–9]. As NMIP solution methods are not robust, their implementations are not robust and cannot consider large power systems. Therefore, the other driving objective in this work is to develop a solution method for the settlement scheme that is practical and computationally efficient.

In order to address these concerns, this paper proposes a new reactive power market clearing scheme. It considers generators, transmission system elements such as reactive power sources and transformers. Transformers are remunerated based upon their action of tap shifting. A non-linear mixed integer programming formulation is developed. It is solved using a proposed successive mixed integer linear programming (MILP) technique. The use of MILP lends robustness to the proposed solution method. Tests on IEEE 6-bus and 300-bus systems are completed and presented in the paper with a discussion on effective prices.

2. Market structure and economic costs of reactive power generation and transmission

2.1. Market structure

The market structure used in this paper follows the one proposed in [8]. It uses a *monopsonic* market structure. Reactive power is purchased from all the sellers by a single buyer, the ISO. In this case, the price is supplied by the sellers to the ISO. The ISO settles the market by solving the optimization problem proposed and solved in this paper.

The formulation assumes that the real power market has been settled and that real power generation has been decided. This allows determination of opportunity costs. The ISO receives reactive power cost curve coefficients from the vendors a day-ahead and settles the market a day-ahead.

The elements that participate in the generation and transfer of reactive power are analyzed and priced for their services. These include generators, transmission system elements such as reactive power sources and transformers. The pricing scheme for the service rendered by a transformer is based upon its action of tap shifting. This section presents the pricing scheme for each participating element.

2.2. Cost of reactive power supply from generators

The formulation assumes that the real power market has been settled and that real power generation has been decided. Cost of reactive power supply from a generator includes costs associated with losses in its field circuit, stator circuit and *lost opportunity cost*. Lost opportunity cost of a generator is computed based on its real power schedule, capability chart and other considerations. Lost opportunity costs are not applicable for synchronous condensers. Aggregating the above costs and based on the knowledge of a synchronous generator's operating characteristics, the cost curve of its reactive power supply is shown in Fig. 1 [8]. It may be noted that the four segments of this graph are linear. A non-linear cost curve can be approximated to this form or by adding more segments if warranted.