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Uncertainty analysis of power system state estimates and reference bus selection

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ABSTRACT

The state estimation (SE) in an electric power system is subject to uncertainty, due to random measurement errors, inaccurate line parameters, etc. Specifically, the location of the reference bus may influence over the accuracy of some estimated states. This paper analyzes this effect and proposes an algorithm to select the reference bus, pursuing maximum overall accuracy. Two realistic case studies are analyzed and conclusions duly drawn.

1. Introduction

1.1. Motivation

A secure and efficient operation of any electric power system can only be achieved if the Control Center has access to accurate information of the status of the system at any time. This information is provided by the state estimator. The main purpose of the state estimator is to compute the most-likely state of the power system, given a redundant set of measurements. Additionally, the state estimation procedure allows the detection and identification of bad measurements, the detection of topology errors, and the detection of inaccurate values for the model parameters.

Due to random measurement errors, any resulting estimate is subject to an estimation error, i.e., the estimate usually does not exactly coincide with the unknown true value. Since the measurements can be modeled by statistical distribution functions, the estimates can also be modeled by distribution functions. Considering that these distributions are Gaussian, each estimate can be characterized by its mean and variance. This variance measures the dispersion of the range within which it can be assured that the unknown true state lies.

The location of the reference bus may have a significant impact over the quality of the estimated voltage angle states. In fact, a large amount of works in the SE technical literature employ the voltage magnitude/angle states to evaluate the estimation accuracy obtained. As a consequence, a change of the reference bus may affect those results obtained. For this reason, in this paper a variance analysis of the state estimates is performed and a method to allocate the reference bus is proposed, pursuing a higher overall estimation accuracy. It should be noted that the selection of the reference bus does not affect neither the estimated active/reactive power flows nor the estimated voltage magnitudes.

1.2. Literature review

In power systems, the concept of state uncertainty was first used by Schweppe [1], and later applied to other areas, such as hydro systems [2]. Reference [3] develops an uncertainty analysis for power system state estimation, proposing a method to compute the confidence interval of the estimates based on a linear optimization problem. Work [4] improves and generalizes the method in [3], developing an algorithm to compute the confidence bounds of the estimates considering uncertainty in both measurements and parameters, leading to a sequential quadratic optimization problem. Recently, the method proposed in [3] has been improved in [5], and solved using constrained nonlinear optimization and Monte Carlo simulation. A similar uncertainty analysis has been also applied to power flow studies in [6], using an affine arithmetic-based methodology.

On the other hand, a significant number of works check the estimation accuracy of their methods employing metrics based on the voltage magnitude/angle state estimates. These works address different aspects concerning the SE, such as meter placement algorithms [7,8], SE in distribution networks [9–11], inclusion of PMUs

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in the measurement set [12–16], implementation of heuristics or artificial neuronal network techniques [10,17], and multiarea and robust estimation methods [18–21], among others. In the aforementioned works, the numeric results obtained may vary if the reference bus is relocated in other bus of the corresponding system studied.

To the best of author's knowledge, no prior work has analyzed the effect of the reference bus selection over the accuracy of the estimated states.

The location of the slack bus in the framework of power flow analysis has been deeply studied by the technical literature. Some works develop the distributed slack bus concept, developing algorithms based on participation factors [22–25]: such as a highly decoupled load flow method [22], the computation of system incremental costs [24], or the computation of reference, congestion and loss prices [25]. Other works provide a slack bus location method in order to reduce the power imbalance [26] or even a technique to remove the slack bus maintaining an equal incremental cost [27].

1.3. Contribution

The contribution of this paper is twofold. First, using numerical simulations, it is shown that the uncertainty of the voltage angle estimates significantly varies depending on the selection of the reference bus. Second, three methods are developed to optimally select the reference bus with the aim of minimizing the average variance of the estimated states. The robustness of the proposed algorithm is also analyzed.

1.4. Paper structure

The rest of this paper is organized as follows. Section 2 briefly describes the state estimation technique, and numerically shows that the variance of the voltage angle estimates depends on the selection of the reference bus. In Section 3, three algorithms are described to optimally select the reference bus, based on extensive search and local search methods. In Section 4, a robustness study is performed, examining the robustness of the solutions of the proposed algorithms. In Section 5, the developed algorithms are tested using the IEEE 118-bus and the 300-bus systems, and their performances are compared. Finally, conclusions are presented in Section 6.

2. State estimation

Almost any state estimator can be formulated as an optimization problem using the Weighted Least Squares approach [28]:

$$\underset{\boldsymbol{x}}{minimize} \quad [\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x})]^T \boldsymbol{R}_z^{-1} [\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x})] \tag{1}$$

where $\boldsymbol{x}$ is the $(n \times 1)$ state vector, $\boldsymbol{z}$ is the $(m \times 1)$ measurement vector, and $\boldsymbol{R}_z$ is the covariance matrix of measurements. Vectors $\boldsymbol{x}$ and $\boldsymbol{z}$ are related by the equation $\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}^{\text{true}}) + \boldsymbol{e}_z$, where $\boldsymbol{h}(\cdot)$ is a multifunctional vector, $\boldsymbol{x}^{\text{true}}$ is the unknown true system state, and $\boldsymbol{e}_z$ is the unknown measurement error vector.

In this work, it is assumed that each measurement error is an independent unbiased Gaussian-distributed random variable, and cross correlations between measurement errors are disregarded, i.e., $\boldsymbol{R}_z$ is considered a diagonal matrix. Thus, $\boldsymbol{R}_z = \text{diag}(\sigma_{z1}^2, \sigma_{z2}^2, \ldots, \sigma_{zm}^2)$, where σ_{zi}^2 is the variance of the i-th measurement. The proposed procedure is general, and it can be straightforwardly expanded to consider non-Gaussian and/or correlated input measurements.

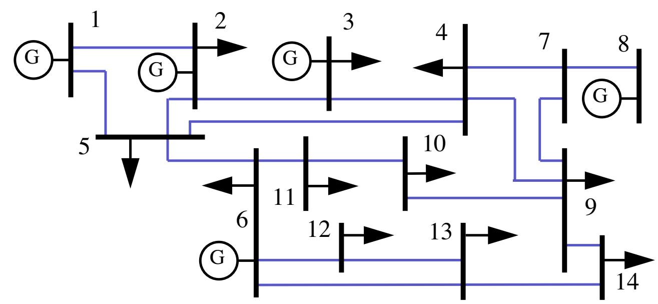

Fig. 1. IEEE 14-bus system.

The optimal solution $\hat{\boldsymbol{x}}$ of problem (1) can obtained as the limit of a sequence of states $\hat{\boldsymbol{x}}_\nu$ by means of a Newton-based recursive algorithm, one step of which is:

$$\hat{\boldsymbol{x}}_{\nu+1} - \hat{\boldsymbol{x}}_\nu = (\boldsymbol{H}^T(\hat{\boldsymbol{x}}_\nu)\boldsymbol{R}_z^{-1}\boldsymbol{H}(\hat{\boldsymbol{x}}_\nu))^{-1}\boldsymbol{H}^T(\hat{\boldsymbol{x}}_\nu)\boldsymbol{R}_z^{-1}[\boldsymbol{z}-\boldsymbol{h}(\hat{\boldsymbol{x}}_\nu)] \tag{2}$$

where ν corresponds to the iteration number.

The optimal solution is $\hat{\boldsymbol{x}} = [\hat{V}_1 \ldots \hat{V}_n \quad \hat{\theta}_1 \ldots \hat{\theta}_n]^T$; $\hat{V}_i$ and $\hat{\theta}_i$ are the estimated values of voltage magnitude and angle at the ith bus, respectively; and matrix $\boldsymbol{H}(\cdot)$ is the Jacobian of $\boldsymbol{h}(\cdot)$. The optimal solution does not include the reference angle as an element of the optimization variable vector.

To deal with non-Gaussian input variables, the method described in [29] can be applied: Rosenblatt and Nataf transformations are employed to compute an alternative weighting matrix, to be used in Eq. (1). The iterative algorithm to obtain the optimal estimated state (Eq. (2)) and the proposed method described in Section 3 are valid.

If input variables are correlated, those cross-correlations are modeled by using a block-diagonal weighting matrix in (1), as described in work [30]. These non-diagonal elements in $\boldsymbol{R}_z$ (corresponding to second-order cross-moments) can be computed using either the point estimate method [31] or the unscented transformation [32]. Again, the Newton-Raphson algorithm in (2) and the developed methodology described in Section 3 are valid.

2.1. Estimation accuracy and estimation variance

The accuracy of an estimate $\hat{\boldsymbol{x}}$ can be expressed as the "distance" between $\hat{\boldsymbol{x}}$ and $\boldsymbol{x}^{\text{true}}$. The higher the distance, the less accurate the estimation is. As it is customary, we assume that the estimate is close enough to the true state:

$$\text{E}[\hat{\boldsymbol{x}}] \simeq \boldsymbol{x}^{\text{true}} \quad . \tag{3}$$

However, it is important to consider also the *variability* of the estimate. The variability of an estimate denotes the dispersion of the range of the estimation value, and it can be quantified by means of the estimation variance. The higher the estimation variance, the higher the dispersion of the range of the estimate.

For example, let us consider two estimates: $\hat{x}_1$ and $\hat{x}_2$. Let us assume that the variances of these estimates are $\sigma_{\hat{x}_1}^2$ and $\sigma_{\hat{x}_2}^2$, respectively, and that $\sigma_{\hat{x}_1}^2 < \sigma_{\hat{x}_2}^2$. Then, although $\text{E}[\hat{x}_1] \simeq x_1^{\text{true}}$ and $\text{E}[\hat{x}_2] \simeq x_2^{\text{true}}$, it is very likely that $|\hat{x}_1 - x_1^{\text{true}}| < |\hat{x}_2 - x_2^{\text{true}}|$. Note that $|\hat{x}_2 - x_2^{\text{true}}|$ is higher than $|\hat{x}_1 - x_1^{\text{true}}|$ because the dispersion of $\hat{x}_2$ is higher than that of $\hat{x}_1$. Thus, the state variable x_2 is estimated with more uncertainty than the state variable x_1. i.e., the estimation of variable x_2 is less accurate.

For illustrative purposes, the IEEE 14-bus system [33] is considered (see Fig. 1). For a given measurement location configuration, one thousand measurement scenarios are generated using random Gaussian-distributed errors added to the known true state $\boldsymbol{x}^{\text{true}}$ computed from a power flow solution. For each measurement