Electrical Power and Energy Systems 54 (2014) 442–453

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Direct approach in computing robust Nash strategies for generating companies in electricity markets

Damoun Langary [a], Nasser Sadati [a,b,*], Ali Mohammad Ranjbar [b]

[a] Intelligent Systems Laboratory, Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
[b] Center of Excellence in Power System Management and Control (CEPSMC), Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

A R T I C L E I N F O

Article history:
Received 7 February 2013
Received in revised form 24 July 2013
Accepted 25 July 2013

Keywords:
Game theory
Supply function equilibrium (SFE)
Strategic bidding
Nash equilibrium (NE)
Electricity market

A B S T R A C T

Supply function equilibrium (SFE) is often used to describe the behavior of generating companies in electricity markets. However, comprehensive analytical description of supply function models is rarely available in the literature. In this paper, using some analytical calculations, a novel direct approach is proposed to compute the Nash equilibrium (NE) of the supply function model under uniform marginal pricing mechanism. An explicit mathematical proof for its existence and uniqueness is also presented. The proposed methodology is then generalized to accommodate practical market constraints. In addition, a new concept of robust NE is introduced and calculated based on this approach. Finally, numerical simulations demonstrate the applicability and effectiveness of the proposed solution scheme.

1. Introduction

In recent years, the trend of electricity industry in many countries has been towards a less regulated and more competitive energy market. Several methods and theories have been introduced and utilized to model different aspects of deregulated pool-based electricity markets. Agent-based modeling [1–3], statistical methods [4,5], artificial intelligence based approaches [6–8], optimization theory [9], and of course game-theoretic approaches [10–14] have been all frequently used in the literature. Among these approaches, game theoretic methods not only have better ways of realistically simulating the oligopolistic competition in electric power markets, but also are perfectly applicable to a wide range of market models such as Bertrand, Cournot, Stackelberg and supply function models [15–17].

Among all aforementioned market models, the Cournot and SFE models seem to fit better and hence are more frequently exploited in the analysis of electricity markets. The Cournot models are simpler in essence and provide more intuition into the actual behavior of the market through analytical results. On the other hand, supply function models grant much more consistency with what really goes on in most electricity markets, and as a result, many of recent publications in the literature have implemented SFE in their simulations. In spite of certain well-known advantages of the supply function models [18], there are some known limitations in properly applying SFE to analyze strategies of generating companies in power markets [19].

The first and probably most important drawback of supply function models is the uniqueness issue; it is straightforward to show that an infinite number of NEs could exist [20,21]. To tackle this issue, Baldick [21] and others have examined some arbitrary parameterization methods to restrict the results to a unique NE point, which is frequently used in the literature. This simplification process of course leads to some other questions on how to set those parameters. Another downside to supply function models is that the NE is usually calculated through some iterative algorithms that not only do not have any guarantees of convergence, but also provide little insight into the characteristics of the calculated NE.

In this paper, we present a new approach to directly calculate the NE of electricity markets using the supply function model. The solution is then generalized to support all aforementioned parameterization methods [21]. This direct computation approach not only guaranties the existence and uniqueness of the NE, but also opens way to analyze the effect of parameterization techniques in the resulted equilibrium point. Given this new perspective into the NE calculation problem, we then proceed to some more advanced ideas to obtain a more eligible assessment of independent power producers' behavior according to the characteristics of the actual market. As it will be shown, the proposed method is also applicable to computing NE of Cournot models.

* Corresponding author at: Center of Excellence in Power System Management and Control (CEPSMC), Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran. Tel.: +98 (21)6616 4365; fax: +98 (21)6616 5939.
E-mail addresses: sadati@sharif.ir, sadati@ece.ubc.ca (N. Sadati).

0142-0615/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijepes.2013.07.031

The remainder of this paper is structured as follows. Section 2 contains the literature review. The problem formulation is given in Section 3. In Section 4, the proposed solution method is presented and discussed in detail. The method is then generalized in Section 5, to cope with existent constraints of the actual markets. In Section 6, the new concept of robust Nash strategy is introduced and resolved. The simulation studies are presented in Section 7. Finally, concluding remarks are drawn in Section 8.

2. Literature survey

The concept of SFE was originally developed by Klemperer and Meyer [20] as a better way of modeling independent players' competing behaviors in markets under uncertain demand conditions. The SFE approach was later adopted by Green and Newberry [22,23] as a model for strategic bidding in the British and Wales' electricity spot market, assuming that suppliers must bid the same supply function across multiple pricing periods. Their researches, however, implied some oversimplifying restrictions on the cost functions of the participating suppliers. In addition, they did not explain how to incorporate transmission constraints into their approach. Using similar restricting assumptions of [1], Rudkevich proved that if the players begin the bidding based on their marginal costs and then update their strategies according to a described learning process, in the limit, the supply functions converge to a SFE which provides an explanation of how might the bids of actual players evolve into their Nash strategies. A detailed literature review of bidding strategies in electricity markets are presented in [24].

In a distinguished work in 2002 [21], Baldick compared the main variations of supply function models of bid-based electricity markets in presence and absence of transmission constrains. Based on artificial restrictions that are arbitrarily placed on the strategic parameters of players, he classified the supply function models used in literature into four categories, namely, *R – parameterization*, *c – parameterization*, $(R \propto c)$ – *parameterization* and (R, c) – *parameterization*. Whilst the latter parameterization method indicates the true nature of supply function models, the restricted models are more frequently used because of their simplicity. Baldick concluded that parameterization of the SFE has a significant effect on the calculated results, to such an extent that several SFE results demonstrated in the literature are, in fact, artifacts of assumptions about the choices of particular bid parameters.

de la Torre et al. [25,26] presented a detailed model of the electricity market consisted of profit-maximizing generating companies (GENCOs), by taking multi-period bidding, price elasticity of demand and network modeling into account. The behavior of market participants and the market itself were characterized through a repetitive simulation procedure. Each firm, using a price-quota curve, is able to solve its profit maximization problem and construct its optimal bidding strategy. The method can be of interest to regulators that study the behavior of oligopolistic markets and also to GENCOs who want to know the best strategy to follow.

In [27], the authors extended the proposed idea of [10] to develop a more general method for analyzing the competition among transmission-constrained GENCOs with incomplete information. Each GENCO models its opponents' unknown information with specific types to transform the game of incomplete information into a complete game with imperfect information. Using SFE to model GENCOs' bidding strategies, the competition was modeled as a bi-level optimization problem: the upper subproblem represents the GENCOs' profit maximization and the lower subproblem is the ISO's market clearing problem, which is solved by minimizing consumers' total payment as a measure of social welfare. Price elasticity of demand was assumed to be zero in this study. Using GAMS programming language, the same bi-level problem was solved in [28], in which the authors used a similar iterative algorithm to find the NE, and thus, the bidding strategies of GENCOs in a day-ahead energy market. In [29], a similar bi-level optimization problem was solved to find the NE using MPEC approach.

In [30], using analytical computations, the authors have provided closed formulas for price, quantities and profits for a short-term electricity market whose participants rely on Cournot models. Based on those formulas, the case of several identical Cournot GENCOs and the case of one dominant GENCO were compared in a detailed manner. In [31], the authors have proposed a new algorithm for the calculation of Cournot equilibrium in the absence of transmission constraints, and hereby analyzed and investigated the properties of deregulated electricity markets using game theory. The algorithm is based on transforming the game into a three level decision making process with economic signal exchange. They argued that even though SFE models are better representatives of actual biddings in electricity markets, they do not possess the attractive features of the Cournot models as far as ease of modeling and eligible computation of NE are concerned. The main drawback of Cournot models, however, is their high level of sensitivity to the price elasticity of demand curve.

In [13], Bompard et al. have provided a comparative analysis of the application of game theoretic models to simulate the oligopolistic competition in network constrained electricity markets, by focusing on strategic behavior of electricity producers. Several models such as SFE, Cournot, Stackelberg and conjectural supply function models were considered and their appropriateness to model the electricity was discussed. Furthermore, the effects of both strategic bidding and network constraints on the efficiency of the market were investigated.

3. Problem formulation: linear SFE

Assuming that independent power producers (IPPs) are rational self-interested players, the objective of each player is to maximize its respective profit function. Since each player's profit not only depends on its own bidding strategy, but also on its rivals' bidding behavior, the bidding strategies of GENCOs and the NE of the market needs to be determined at once. The market clearing mechanism is assumed to have uniform pricing.

Assuming a linear supply function model for GENCOs' bidding strategies, we have

$$\rho = \alpha_i P_i + \beta_i; \quad i = 1, \ldots, n_G \tag{1}$$

where ρ is the market clearing price (MCP) and P_i is the production level of ith player (GENCO). Also α_i and β_i present the strategic bidding parameters of ith player in the supply function model. Each player, willing to increase its own profit, might be able to do so either by increasing the MCP using higher values of strategic parameters α_i and β_i (offering higher prices for the same amount of electric energy) or by increasing its output via decreasing its strategic parameters (offering higher amounts of electric energy for the same price). The objective of each GENCO is then to find a profit-maximizing supply function, by tuning its control variables α_i and β_i via an optimization process.

The demand function of the market is assumed to be elastic (price-sensitive), expressed as follows

$$D = D_0 - \gamma \cdot \rho \tag{2}$$

where D is the actual demand level at market price ρ. D_0 is an initial demand value and γ is a price elasticity coefficient, accounting for a mild level of elasticity in demand of electric power markets.

Now, by introducing new control variables $\omega_i = 1/\alpha_i$ and $\theta_i = \beta_i/\alpha_i$ for all players, Eq. (1) can be rewritten as follows

$$P_i = \omega_i \rho - \theta_i; \quad i = 1, \ldots, n_G \tag{3}$$