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Analysis and design of reduced order linear quadratic regulator control for three phase power factor correction using Cuk rectifiers

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ABSTRACT

In this paper, the analysis and design of reduced order linear quadratic regulator (ROLQR) control for power factor correction (PFC) in a three phase system is presented. The front end is a three phase diode rectifier followed by DC–DC Cuk converter modules with the common DC output. Instantaneous symmetrical component theory is used for the generation of reference current. The control strategy uses three inner ROLQR current controllers for source current shaping and an outer voltage loop using PI controller for load voltage regulation. It uses a single stage converter for both PFC and voltage regulation. The proposed method offers simple control strategy, fast transient response and power factor closed to unity. This type of three-phase three switch PFC converter features a simple and robust configuration compared to conventional six switch topology. To validate the proposed method, a prototype controlled by dSPACE 1104 signal processor is set up. Simulation and experimental results indicate that the proposed system offers regulated output voltage for wide load variations and power factor close to unity.

1. Introduction

Recently, there is growing awareness about line pollution and deteriorating power factor due to the usage of pervading inductive and non-linear loads. Although many solutions were offered for single phase power factor correction (PFC), three phase active PFC was seldom considered. As all high power equipment derive electrical power from three phase mains, incorporating an active three phase PFC front end can contribute significantly in improving the overall power factor and reducing line pollution. Many literature have been proposed for PFC. A three-phase single switch PFC topology has the merits of simple control and few components [1–5]. This type of converter suffers due to Discontinuous Conduction Mode (DCM) operation, causing high current stresses on the power devices. A three phase six switch Pulse Width Modulation (PWM) boost rectifier is widely used for high-power applications. Advantages of this type of rectifier are high efficiency and good current quality. But the PWM rectifier needs a complex control structure and the efficiency is lower than the diode rectifier due to additional switching losses [6–13]. A three-phase three-switch topology composed of three single-phase single-switch modules was proposed for PFC in [14,15]. Even though the above method offers simple control implementation, it fails to operate in case of one or two module failures. A derived version of buck boost rectifier is a Cuk rectifier that inverts the voltage polarity and can also simultaneously increase or decrease the voltage magnitude. It has excellent features such as capacitive energy transfer, magnetic components integrability, full transformer utilization and good steady-state performance. It also provides smooth input and output currents due to the presence of inductors in the input and output side [16].

Many methods for generating the reference template were proposed. In [17–19] the instantaneous reactive power theory was proposed (i.e. p–q theory) for calculating the reference currents. The general equation for deriving the reference current which relates the new concept of instantaneous active and reactive theory was reported in [20], but no detailed information was given for DC bus voltage compensation. In [21], the extended p–q theory was proposed to derive a more general vector equation for calculating the reference currents. This method does not operate satisfactorily for module loss. In [22], only the final formulation of the extracted reference current was reported. In [23] and [24], PFC using Cuk rectifier modules was proposed and reference current was generated using power balance control technique. The extraction of reference current based on instantaneous symmetrical component theory involves simple computations based on the instantaneous

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source voltages and currents and it does not require any three phase to two phase conversions.

The classic Linear Quadratic Regulator (LQR) approach deals with the optimization of a cost function or performance index. Thus, the designer can weigh which states are more important in the control action to seek for appropriate performance. This feature of LQR control has initiated several researchers to successfully apply this technique in the field of power electronics. In [25] and [26], the performance indices are selected using pole placement technique. This method depends on the exact placement of closed loop poles. In [27], the cost function is derived from an initial controller by frequency domain method which is a time consuming procedure. Optimal control provides a systematic way of designing a LQR controller and problem of where to place the closed loop poles does not arise and is robust to parameter and load variations.

Hence, it is proposed to develop a single stage three phase AC–DC converter using Cuk rectifier modules based on ROLQR control for achieving voltage regulation and PFC. The instantaneous symmetrical component theory is used for calculating the reference currents. To reduce the complexity in analysis, a reduced order model of Cuk converter is obtained from the higher order state space model [28,29] by Pade's approximation technique. The outer voltage loop compensator is designed using Ziegler Nichol's tuning procedure.

2. Modeling of DC–DC Cuk converter

Fig. 1(a) represents the circuit diagram of the Cuk converter. It consists of two inductors L_i, L_o and two capacitors C_t, C_o, V_g and V_o represent supply and output voltage, respectively, S is an active switch, D is a freewheeling diode and R_L is the load resistance. S operates at a switching frequency f_s with duty ratio d.

To obtain the mathematical model of the controller, the state model of Cuk converter is derived by considering $S = 1$ during the MOSFET switch conduction subinterval and $S = 0$ during the diode conduction subinterval. The converter dynamics is described by state–space averaging method and by using the same method, the state equations during switch-on and switch-off conditions are combined as follows:

$$\frac{dx_1}{dt} = -\frac{(1-d)}{L_i}x_3 + \frac{V_g}{L_i} \tag{1}$$

$$\frac{dx_2}{dt} = \frac{d}{L_o}x_3 - \frac{1}{L_o}x_4 \tag{2}$$

$$\frac{dx_3}{dt} = \frac{(1-d)}{C_t}x_1 - \frac{d}{C_t}x_2 \tag{3}$$

$$\frac{dx_4}{dt} = \frac{1}{C_o}x_2 - \frac{1}{R_L C_0}x_4 \tag{4}$$

where x_1, x_2, x_3, x_4 are the current through the inductor (i_{Li}), current through the inductor (i_{Lo}), voltage across the transfer capacitor (v_{ct}), voltage across the output capacitor (v_{co}), respectively, and d represents the duty cycle. From Eqs. (1)–(4), the averaged system matrices are derived as given below:

$$A = \begin{bmatrix} 0 & 0 & \frac{-(1-d)}{L_i} & 0 \\ 0 & 0 & \frac{d}{L_o} & -\frac{1}{L_0} \\ \frac{(1-d)}{C_t} & -\frac{d}{C_t} & 0 & 0 \\ 0 & \frac{1}{C_o} & 0 & -\frac{1}{R_L C_o} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_i} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

2.1. Design of DC–DC Cuk converter components

A Cuk PFC was designed [23] with the following specifications: rectified input voltage $V_g = 24$ V, line frequency 50 Hz, power factor ≥ 0.99, maximum output power $P_o = 56$ W, $R_L = 10\,\Omega$, output DC voltage $V_o = -24$ V, efficiency $\eta \geq 85\%$. DC voltage conversion ratio M is given by

$$M = \frac{V_o}{V_g} = 0.704 \tag{6}$$

Conduction parameter of the Cuk PFC circuit

$$K_a > \frac{1}{(2(M + |\sin(\theta)|)^2)} > 0.1752 \tag{7}$$

An equivalent inductance is given by

$$L_{eq} = \frac{R_L T_S K_a}{2} = 0.667 \text{ mH} \tag{8}$$

By choosing the input inductor current with 6.75% current ripple, $\Delta i_{Li} = 0.169$ A. The design of L_i and L_o is made using the desired ripple value of the input current. For the duty ratio $d = 0.5$, switching frequency = 50 kHz, switching period $T_s = 0.25\,\mu$s, the input inductor value is found to be

$$L_i = \frac{V_g d T_S}{\Delta i_{Li}} = 2 \text{ mH} \tag{9}$$

The output inductor value is found to be

$$L_o = \frac{L_1 L_{eq}}{L_1 - L_{eq}} = 1 \text{ mH} \tag{10}$$

Transfer capacitor value is calculated by considering a resonant frequency of 1 kHz and is given by

$$C_t = \frac{1}{\omega_{ro}^2 (L_i - L_o)} = 25.35 \ \mu\text{F} \tag{11}$$

To select the output capacitance value in this circuit, the main factor is the output ripple voltage which is caused by the second harmonic. The equation to define the output capacitance value can be expressed as

$$C_o = \frac{P_{in}}{2\pi f_r V_o \Delta V_o} = 2211 \ \mu\text{F} \tag{12}$$

where ΔV_o is the peak value of the output ripple voltage = 0.6 V (2.5% of the output voltage) and f_r is the second harmonic frequency. The design values are substituted in (5) and after applying phase variable transformation, A and B matrices become,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6.3 \times 10^{12} & -1.87 \times 10^{9} & -3.8 \times 10^{7} & -50 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{13}$$

The main objective is to shape the input source current and to regulate the output voltage of the DC–DC Cuk converter. The converter requires sensing of four state variables, which is not acceptable from practical point of view. In order to reduce the complexity in controller design, the fourth order model of a Cuk converter is reduced to a second order model. The model reduction technique used is Pade's approximation, wherein the two dominant poles of the system i.e. the input inductor current, i_{Li} and the output capacitor voltage, v_{Co} are retained and the effects of the transfer capacitor C_t, and the output inductor L_o are neglected. Hence it becomes sufficient to regulate these two variables i_{Li}, v_{Co} using the ROLQR control strategy. The reduced order matrix for the above system is given as

$$A = \begin{bmatrix} 0 & 1 \\ -1.64 \times 10^5 & -49.34 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{14}$$