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Calculation of reactivity and power factors depending on the external source location

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A B S T R A C T

We used the neutron diffusion equation with external neutron sources, in cartesian geometry and the two groups of energy, to verify the influence of external neutron source locations in the calculation of reactivity and power factors. To this end, the Coarse Mesh Finite Difference (CMFD) method was applied to the adjoint flux calculation and to simplify reactivity calculation in PWR type reactor, using the output of the Nodal Expansion Method (NEM). Different locations on the two-dimensional plane, as well as different types of fuel elements in the reactor core were used in the present study.

1. Introduction

The main objective of this paper was to evaluate if the location of the external neutron source directly affects the calculation of the reactor core reactivity and power factors. We know that the external neutron sources are extremely important during the reactor start-up, because in this subcriticality range they are responsible for signal fluctuation reduction of source and intermediary range detectors. Sources can be classified as primary sources such as ^{252}Cf (Californium), generally used on the first nuclear reactor loading, with a neutron emission rate of approximately 2.3×10^{12} neutrons/s/g and a relative high half-life of approximately 2.65 years and secondary sources as ^{124}Sb (Antimony) and $^{9}_{4}$Be (Beryllium), generally used after the first cycle for neutron generation in the reactor that emit approximately 2.7×10^{9} neutrons/s/g with relative low half-life of approximately 60 days.

In order to evaluate the influence of the location of the external source we used different positions in the two-dimensional core. The Nodal Expansion Method (NEM) (Finnemann et al., 1977) was used to obtain the neutron flux for a known configuration of control rods banks. Reactivity was obtained from the results of NEM used as input in the Coarse Mesh Finite Difference (CMFD) Method (da Silva et al., 2011).

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2. Neutron diffusion equation with external neutron source

The coarse mesh nodal methods, such as NEM, divide the volume of the reactor core into a set of contiguous parallelepipeds, named nodes, in which the nuclear multigroup parameters, as well as the neutron cross sections, are uniform. One of the main characteristics of this method is that it uses the transverse-integrated diffusion equation, generating a set of three one-dimensional equations coupled through transverse leakage terms, the solution of which supplies a relation between average fluxes and average net currents on the faces of the node. The starting point of this method is the modified neutron continuity equation, which integrated in volume $V_n = a_x^n a_y^n a_z^n$ of node n, for two energy groups is given by:

$$\sum_{u=x,y,z} \frac{1}{a_u^n}\left[\bar{J}_{gur}^n - \bar{J}_{gul}^n\right] + \Sigma_{rg}^n \bar{\phi}_g^n = \sum_{\substack{g'=1 \\ g' \neq g}}^{2} \Sigma_{gg'}^n \bar{\phi}_{g'}^n + \chi_g \sum_{g'=1}^{2} \nu \Sigma_{fg'}^n \bar{\phi}_{g'}^n + \bar{S}_g^n; \quad g = 1,2, \quad (1)$$

where the terms are defined as: $\bar{J}_{gus}^n$: neutron net current of group g on face $s = l, r$, in direction u; Σ_{rg}^n: macroscopic removal cross section of group g; $\Sigma_{gg'}^n$ macroscopic scattering cross section from group g' to group g; χ_g: fission spectrum of group g; $\nu\Sigma_{fg'}^n$: product of the average number of neutrons emitted by fission by the macroscopic fission cross section of group g'; $\bar{S}_g^n$: external neutron source of group g; $\overline{\phi}_g^n$: neutron flux of group g; a_u^n: size of node n in direction u.

The neutron flux can be obtained from equation (1), stating the intensity of the external neutron source and using the average multigroup nuclear parameters in the nodes.

This equation can be written in the following matrix form for two energy groups (Pereira et al., 2005),

$$A\Phi = F\Phi + S, \tag{2}$$

where

$$A \equiv \begin{pmatrix} -\nabla\cdot(D_1^n\nabla) + \sum_{r1}^n & 0 \\ -\sum_{21}^n & -\nabla\cdot(D_2^n\nabla) + \sum_{r2}^n \end{pmatrix},$$

and

$$F \equiv \begin{pmatrix} \chi_1\nu\sum_{f1}^n & \chi_1\nu\sum_{f2}^n \\ 0 & 0 \end{pmatrix}.$$

The neutron flux in reactor Φ and the external source S are expressed as column vectors in the following way:

$$\Phi \equiv \begin{pmatrix} \overline{\phi}_1^n \\ \overline{\phi}_2^n \end{pmatrix}, \quad S \equiv \begin{pmatrix} \overline{S}_1^n \\ 0 \end{pmatrix}.$$

Specifying the intensity of the external neutron source, the neutron flux in a subcritical system can be obtained from the solution of equation (1). This calculation is usually known as "fixed source problem" (Duderstadt and Hamilton, 1976).

3. Method for reactivity determination

In a subcritical system with control rods banks partially inserted in the reactor core, equation (2) may be re-written as follows,

$$(A + \delta A)\varphi = (F + \delta F)\varphi + S, \tag{3}$$

where δA and δF represent the perturbations due to the insertion of the control rods banks of matrices A and F, defined at Section 2 (da Silva et al., 2011).

Multiplying equation (3) by the transpose adjoint flux of the critical problem $\phi^{\dagger T}$ and integrating over the entire reactor volume we have,

$$< \phi^{\dagger T}, (A - F)\varphi > = < \phi^{\dagger T}, (\delta F - \delta A)\varphi > + < \phi^{\dagger T}, S >. \tag{4}$$

After certain algebraic manipulations, we obtain the following definition of reactivity (Greenspan, 1975):

$$\rho \equiv -\frac{< \phi^{\dagger T}, (\delta F - \delta A)\varphi >}{< \phi^{\dagger T}, (F + \delta F)\varphi >} = -\frac{< \phi^{\dagger T}, S >}{< \phi^{\dagger T}, (F + \delta F)\varphi >}, \tag{5}$$

where ρ is the reactivity calculated for the system for different configurations of control rods banks inserted in the reactor core, $\phi^{\dagger T}$ the mathematical transpose adjoint flux, F neutron fission term and δA and δF represent the perturbations due to the insertion of the control rods in the reactor core.

4. Power factor calculations

One of the main objectives of general calculations for nuclear reactors is to determine the power distribution in the reactor core, for each fuel element. With the average neutron flux obtained by equation (1), we can calculate the average power density at the node using the following relation:

$$\overline{p}^n = \sum_{g=1}^{2} \gamma_g \Sigma_{fg}^n \overline{\phi}_g^n, \tag{6}$$

where γ_g is the energy produced per fission for the energy group g. The average power density of the reactor core can be determined from the average power density on the node,

$$\overline{P}^{\text{Core}} = \frac{1}{V_{\text{Core}}} \sum_{n=1}^{N} \overline{p}^n V_n, \tag{7}$$

where N is the total number of nodes and V_{Core} the total volume of the reactor core.

In order to verify the influence of the external neutron source location in the calculation of the power distribution in the reactor core, we will calculate the power factor $f_{\text{Power}}^{\text{FE}}$ for each fuel element (FE). Initially, we will determine the average power density in the fuel element $\overline{P}^{\text{FE}}$, defined as the sum of the products of the average power density by the volume of the node, for all the nodes of the fuel element, divided by the volume of the fuel element as follows:

$$\overline{P}^{\text{FE}} = \frac{1}{V_{\text{FE}}} \sum_{n \in \text{FE}} \overline{p}^n V_n = \frac{1}{\sum\limits_{n \in \text{FE}} V_n f^n} \sum_{n \in \text{FE}} \overline{p}^n V_n f^n, \tag{8}$$

where V_{FE} is the total volume of the fuel element, V_n the volume of the node and f^n the contribution factor of the volume of the node (in the case of 1/4 core symmetry), as illustrated in Fig. 1. It is important to enhance, that in the case of an entire core $f^n = 1$ for any node.

Note that for core power calculation, we can define a contribution factor of the fuel element f^{FE} for the core, in such a way that:

$$\sum_{\text{FE}} V_{\text{FE}} f^{\text{FE}} = V_{\text{Core}}, \tag{9}$$

and thus, equation (7) can be written as a function of the average power of the fuel element as follows:

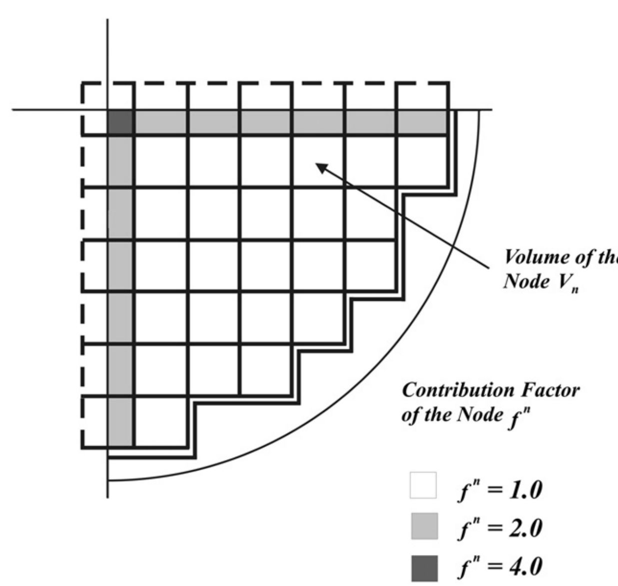

Fig. 1. Contribution factor of the node to the power of the reactor core.