Journal of Economic Behavior & Organization
Vol. 65 (2008) 472–488

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

Evolutionary stability and Nash equilibrium in finite populations, with an application to price competition

Ana B. Ania*

Department of Economics, University of Vienna, Hohenstaufengasse 9, A1010 Vienna, Austria

Received 28 June 2004; accepted 30 December 2005
Available online 26 September 2006

Abstract

Schaffer [Schaffer, M.E., 1988. Evolutionarily stable strategies for a finite population and a variable contest size. Journal of Theoretical Biololgy 132, 469–478] proposed a concept of evolutionary stability for finite-population models that has interesting implications in economic models of evolutionary learning, since it is related to perfectly competitive equilibrium. The present paper explores the relation of this concept to Nash equilibrium in particular classes of games, including constant-sum games, games with weak payoff externalities, and games where imitative decision rules are individually improving. An illustration of the latter is provided in the context of Bertrand oligopoly with homogeneous product which allows for a characterization of the set of evolutionarily stable prices.

JEL classification: B52; C72; D43; D83; L13

Keywords: Bertrand oligopoly; Evolutionary stability; Imitation; Learning; Price competition

1. Introduction

In recent years, a renewed interest in decision rules based on imitation has emerged, partially motivated by the literature on evolutionary game theory. Björnerstedt and Weibull (1996) show that if a game is recurrently played by a continuum population of individuals who mimic the actions of better performing individuals observed at random, population play follows the solution trajectories of the replicator dynamics; imitation is thus one of the possible decision rules underlying the most prominent evolutionary dynamics. It is well known that Nash equilibria are rest points of

* Tel.: +43 1 4277 37433; fax: +43 1 4277 9374.
E-mail address: ana-begona.ania-martinez@univie.ac.at.

0167-2681/$ – see front matter
doi:10.1016/j.jebo.2005.12.002

the replicator dynamics. Moreover, evolutionarily stable strategies (henceforth ESS) as defined by Maynard Smith and Price (1973) for a continuum population are always Nash equilibrium strategies and asymptotically stable in the replicator dynamics. Further, if a state is the limit of a trajectory starting in the interior of the state space, then this limit is necessarily a symmetric Nash equilibrium. In summary, if individuals in a large population mimic successful behavior, when population play converges, it does so to a Nash equilibrium. Hence, evolutionary game theory provides a non-rationalistic foundation to equilibrium play (see Weibull, 1995, Chapters 2 and 3).

The relation between ESS, Nash equilibrium, and the long-run outcomes of imitative dynamics in *finite-population* models is not as well understood. This relation constitutes the main interest of the present paper. In particular, we identify classes of games where a finite-population ESS is always a Nash equilibrium strategy.

The concept of finite-population ESS as defined by Schaffer (1988) is not related to Nash equilibrium in general (cf. Section 3).[1] Stochastic models of evolutionary learning in games that postulate individual behavior driven by imitation also yield somewhat contradictory outcomes. A prominent example is provided by Vega-Redondo (1997), who shows that imitation of successful strategies leads to competitive equilibrium in a Cournot oligopoly. Alós-Ferrer et al. (2000), however, show that imitative behavior leads to Nash equilibrium in a Bertrand oligopoly. The latter holds also in the case of convex costs where there is a large set of Nash equilibria beyond the competitive equilibrium (see Dastidar, 1995).[2] These results show that, in finite-population models, imitative behavior may or may not lead to Nash equilibrium, depending on the type of game. The present paper takes a further step in trying to understand how the properties of imitative rules are related to evolutionary stability of Nash equilibrium in finite-population models.

The results obtained thus far point rather to a more general relation between evolutionary stability, the long-run outcomes of imitative behavior, and perfectly competitive (instead of Nash) equilibrium. In a recent paper, Alós-Ferrer and Ania (2005) study this relation for a class of games that includes the Cournot oligopoly. Their focus is on aggregative games, where payoffs to any player depend on own strategy and an aggregate of all players' strategies. For an aggregative game, aggregate-taking behavior can be defined as payoff maximization disregarding the own effect on the aggregate, which is the analogue to perfectly competitive behavior. It is shown that if the game displays strategic substitutability between own strategy and the aggregate, aggregate-taking behavior has strong evolutionary stability properties. These properties, in turn, imply that aggregate-taking behavior is the long-run outcome of a stochastic learning process where individual decisions are based on imitation of successful strategies and random experimentation. Strategic substitutability between own strategy and the aggregate creates a tension between high relative performance, the dominating force behind evolutionary stability, and high absolute performance, which drives Nash equilibrium; as a consequence, imitation leads away from Nash equilibrium.

In contrast, our main focus here will be on games where imitation is improving, that is where mimicking successful strategies always results in a payoff improvement to the imitator. In such games the conflict between absolute and relative payoff maximization is weakened, and we can

[1] Schaffer (1989) shows, for example, that Nash equilibrium and ESS differ in a Cournot duopoly, and Hehenkamp et al. (2004) show this for rent-seeking games. Tanaka (1999, 2000) has similar results for oligopolies with asymmetric cost functions and differentiated product, respectively.

[2] Note that the results for Cournot and Bertrand competition are compatible only in the particular case of homogeneous product and constant unit costs, where firms competing *à la* Bertrand always price competitively in equilibrium. For the case of increasing marginal costs, it is shown that imitative behavior leads to a subset of the set of Nash equilibria. That subset includes the competitive equilibrium only in certain cases.