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Microeconomics of the ideal gas like market models

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ABSTRACT

We develop a framework based on microeconomic theory from which the ideal gas like market models can be addressed. A kinetic exchange model based on that framework is proposed and its distributional features have been studied by considering its moments. Next, we derive the moments of the CC model (Eur. Phys. J. B 17 (2000) 167) as well. Some precise solutions are obtained which conform with the solutions obtained earlier. Finally, an output market is introduced with global price determination in the model with some necessary modifications.

1. Introduction

Starting with an early attempt by Angle [1,2], a number of models based on the kinetic theory of gases have been proposed to understand the emergence of the universal features of income and wealth distributions (see e.g. Refs. [3–5]). The main focus of those models was to develop a framework that would give rise to gamma function-like behavior for the bulk of the distribution and a power-law for the richer section of the population. The CC-CCM models [6,7] have both of these features. The kinetic exchange model proposed by Dragulescu and Yakovenko [8] and later studied in more detail by Guala [9], produces the gamma function-like behavior for the income distribution. We note that all of these models are generally based on some ad-hoc stochastic asset evolution equations with little theoretical foundations for it. Our primary aim in this paper is to develop a consistent framework from which we can address this type of market model. Here we propose a model based on consumers' optimization which can give rise to those particular forms of asset exchange equations used in Refs. [6,8] as special cases. We then focus exclusively on the asset exchange equations and an analytically simple kinetic exchange model is proposed. Its distributional features are analyzed by considering its moments. The same technique is then applied to derive the moments of the distribution of income in the CC model [6] as well. We find that it provides a rigorous justification for the values of the parameters of the distribution, conjectured earlier in Ref. [10]. A possible extension of the microeconomic settings of the basic model is also studied where we consider the output market explicitly with global price determination.

2. The model

We consider an N-agent exchange economy. Each of them produces a single perishable commodity. Each of these goods is different from all other goods. Money exists in this economy to facilitate transactions (existence of money is not formally

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explained here). Any commodity can enter as an argument in the utility function (see Ref. [11] for a detailed discussion on the *theory of utility*) of any agent. These agents care for their future consumption and hence they care about their savings in the current period as well. Each of these agents is endowed with an initial amount of money (the only type of non-perishable asset considered here) which is assumed to be unity for every agent for simplicity. At each time step, two agents meet randomly to carry out transactions according to their utility maximization principle. We also assume that the agents have a time dependent preference structure. More precisely, we assume that the parameters of the utility function can vary over time [12,13]. In what follows, we analyze the trading outcomes when any two such agents meet in the market at some time-step t.

Suppose agent 1 produces Q_1 amount of commodity 1 only and agent 2 produces Q_2 amount of commodity 2 only and the amounts of money in their possession at time t are $m_1(t)$ and $m_2(t)$ respectively. Since neither of the two agents possesses the commodity produced by the other agent, both of them will be willing to trade and buy the other good by selling a fraction of their own production as well as with the money that they hold. In general, at each time step there would be a net transfer of money from one agent to the other due to trade. Our aim is to understand how the amount of money held by the agents changes over time. For notational convenience, we denote $m_i(t+1)$ as m_i and $m_i(t)$ as M_i (for $i = 1, 2$).

We define the utility functions as follows. For agent 1, $U_1(x_1, x_2, m_1) = x_1^{\alpha_1} x_2^{\alpha_2} m_1^{\alpha_m}$ and for agent 2, $U_2(y_1, y_2, m_2) = y_1^{\alpha_1} y_2^{\alpha_2} m_2^{\alpha_m}$ where the arguments in both of the utility functions are the consumption of the first (i.e. x_1 and y_1) and second good (i.e. x_2 and y_2) and amount of money in their possession respectively. For simplicity, we assume that the utility functions are of the Cobb–Douglas form with the sum of the powers normalized to 1 i.e. $\alpha_1 + \alpha_2 + \alpha_m = 1$, which corresponds to the constant returns to scale property (homogeneity of degree one) [11]. Let the commodity prices to be determined in the market be denoted by p_1 and p_2. Now, we can define the budget constraints as follows. For agent 1 the budget constraint is $p_1x_1 + p_2x_2 + m_1 \leq M_1 + p_1Q_1$ and similarly, for agent 2 the constraint is $p_1y_1 + p_2y_2 + m_2 \leq M_2 + p_2Q_2$. What these constraints mean is that the amount that agent 1 can spend for consuming x_1 and x_2 added to the amount of money that he holds after trading at time $t+1$ (i.e. m_1) cannot exceed the amount of money that he has at time t (i.e. M_1) added to what he earns by selling the good he produces (i.e. Q_1). The same is true for agent 2.

The basic idea behind this whole exercise is that both of the agents try to maximize their respective utility subject to their respective budget constraints and the *invisible hand* of the market that is the price mechanism works to clear the market for both goods (i.e. total demand equals total supply for both goods at the equilibrium prices). Ultimately we will study the money evolution equations in such a situation. Formally, agent 1's problem is to maximize his utility subject to his budget constraint i.e. maximize $U_1(x_1, x_2, m_1)$ subject to $p_1.x_1 + p_2.x_2 + m_1 = M_1 + p_1.Q_1$. Similarly for agent 2, the problem is to maximize $U_1(y_1, y_2, m_2)$ subject to $p_1.y_1 + p_2.y_2 + m_2 = M_2 + p_2.Q_2$. Solving those two maximization exercises by a Lagrange multiplier and applying the condition that the market remains in equilibrium, we get the competitive price vector $(\hat{p}_1, \hat{p}_2)$ as $\hat{p}_i = (\alpha_i/\alpha_m)(M_1 + M_2)/Q_i$ for $i = 1, 2$ (see Appendix A.1).

We now examine the outcomes of such a trading process.

(a) At optimal prices $(\hat{p}_1, \hat{p}_2)$, $m_1(t) + m_2(t) = m_1(t+1) + m_2(t+1)$ and this follows directly from Walras' law [11] saying that if all but one market clears then the rest also has to be cleared. That is, demand matches the supply in all markets at the market-determined price in equilibrium. Since money is also treated as a commodity in this framework, its demand (i.e. the total amount of money held by the two persons after trade) must be equal to what was supplied (i.e. the total amount of money held by them before trade). In any case, an algebraic proof is also given in the Appendix A.2.

(b) We now present the most important equation of money exchange in this model. We make a rather restrictive assumption that α_1 in the utility function can vary randomly over time with α_m remaining constant. It readily follows that α_2 also varies randomly over time with the restriction that the sum of α_1 and α_2 is a constant $(1 - \alpha_m)$. In the money demand equations derived from the above-mentioned problem, we substitute α_m by λ and $\alpha_1/(\alpha_1 + \alpha_2)$ by ϵ to get the following money evolution equations as (see Appendix A.3)

$$\begin{aligned} m_1(t+1) &= \lambda m_1(t) + \epsilon(1-\lambda)(m_1(t) + m_2(t)) \\ m_2(t+1) &= \lambda m_2(t) + (1-\epsilon)(1-\lambda)(m_1(t) + m_2(t)). \end{aligned} \tag{1}$$

For a fixed value of λ, if α_1 (or α_2, see Appendix A.3) is a random variable with uniform distribution over the domain $[0, 1-\lambda]$, then ϵ is also uniformly distributed over the domain $[0, 1]$. It may be noted that λ (i.e. α_m in the utility function) is the savings propensity used in the CC model [6].

(c) For the limiting value of α_m in the utility function (i.e. $\alpha_m \to 0$ which implies $\lambda \to 0$), we get the money transfer equation describing the random sharing of money without savings. This form of transfer equation has been used in Dragulescu and Yakovenko [8], Guala [9] and also in the model proposed later in this paper.

(d) It may be noted that at each time step, the price mechanism works only locally, i.e., it works to clear the markets for two commodities (Q_1 and Q_2) only. The markets considered here are perfectly competitive. Also, the set of competitive equilibria is a subset of the set of Pareto Optimal allocation or in other words, all competitive allocations are Pareto Optimal (see Ref. [11] for the definition of *Pareto Optimality*). Hence, all the allocations achieved through such trading processes are Pareto Optimal. Also, since the exchange equations are not sensitive to the level of production, even if for some reason the level of production alters (due to production shock) the form of the transfer equations will remain the same provided the form of the utility function remains the same.

(e) Ref. [13] also presents a microeconomic framework alongwith an asset evolution equation (see also Ref. [12]). But unlike here, the asset evolution equation for the i-th agent in Ref. [13] depends on his own assets only.