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# A new MILP-based approach for unit commitment in power production planning

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ABSTRACT

This paper presents a complete, quadratic programming formulation of the standard thermal unit commitment problem in power generation planning, together with a novel iterative optimisation algorithm for its solution. The algorithm, based on a mixed-integer formulation of the problem, considers piecewise linear approximations of the quadratic fuel cost function that are dynamically updated in an iterative way, converging to the optimum; this avoids the requirement of resorting to quadratic programming, making the solution process much quicker.

From extensive computational tests on a broad set of benchmark instances of this problem, the algorithm was found to be flexible and capable of easily incorporating different problem constraints. Indeed, it is able to tackle ramp constraints, which although very important in practice were rarely considered in previous publications.

Most importantly, optimal solutions were obtained for several well-known benchmark instances, including instances of practical relevance, that are not yet known to have been solved to optimality. Computational experiments and their results showed that the method proposed is both simple and extremely effective.



## 1. Introduction

The Unit Commitment Problem (UCP) is the problem of deciding which power generating units must be committed/decommitted over a planning horizon (that lasts from 1 day to 2 weeks, generally split into periods of 1 h each). The production levels at which units must operate (pre-dispatch) must also be determined to optimise a given objective function. The committed units must usually satisfy the forecasted system load and reserve requirements, as well as a large set of technological constraints.

This problem has great practical significance because the effectiveness of the schedules obtained has a strong economical impact on power generation companies. Due to this reason and to the problem's high complexity (a prove that it is NP-hard has been given in [1]), it has received considerable research attention. Even after several decades of intensive study, it is still a rich and challenging topic of research.

The proposed optimisation techniques for unit commitment encompass very different paradigms. These range from exact approaches and Lagrangian relaxation to rules of thumb or very elaborate heuristics and metaheuristics. In the past, the combinatorial nature of the problem and its multi-period characteristics have prevented exact approaches from being successful in practice: they resulted in highly inefficient algorithms that were only capable of solving small problem instances with virtually no practical interest. Heuristic techniques, such as those based on priority lists, did not totally succeed either, as they often lead to low quality solutions. Metaheuristics had very promising outcomes when they were first explored. The quality of their results was better than those achieved using well established techniques and good solutions were obtained very quickly. However, some drawbacks can be highlighted when metaheuristics are used. If one considers that the ultimate goal is to design a technique that can be accepted and used by a company, one major drawback of metaheuristics is their dependence on parameter tuning. Parameter tuning is time consuming and the complex tuning procedure requires profound knowledge of the algorithm implemented. Furthermore, accurate tuning is vital for algorithm performance. A second drawback is related to the lack of information that metaheuristics provide in terms of the quality of the solution (i.e., how far it is from the optimal solution). Some proposals have been presented to address these drawbacks; but this still remains an open line of research.

An open issue is related to solution optimality and how it affects individual pay-offs in restructured markets where an independent system operator performs a centralised unit commitment. As stated in [2], only if problems are solved to optimality can one guarantee that units will receive their optimal dispatch and pay-off. Therefore, the design and development of optimisation techniques that provide optimal results to unit commitment problems are of crucial importance.

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**Nomenclature**

*Constants*

| | |
|---|---|
| $T$ | length of the planning horizon |
| $\mathcal{T} = \{1,\ldots,T\}$ | set of planning periods |
| $\mathcal{U}$ | set of units |
| $P_u^{\min}, P_u^{\max}$ | minimum and maximum production levels for unit $u$ |
| $T_u^{\text{on}}, T_u^{\text{off}}$ | minimum number of periods that unit $u$ must be kept switched on/off |
| $r_u^{\text{up}}, r_u^{\text{down}}$ | maximum up/down rates of unit $u$ |
| $D_t$ | system load requirements in period $t$ |
| $R_t$ | spinning reserve requirements in period $t$ |
| $a_u$, $b_u$, $c_u$ | fuel cost parameters for unit $u$ |
| $a_u^{\text{hot}}, a_u^{\text{cold}}$ | hot and cold start up costs for unit $u$ |
| $t_u^{\text{cold}}$ | number of periods after which the start up of unit $u$ is evaluated as cold |
| $y_u^{\text{prev}}$ | previous state of unit $u$ (1 if on, 0 if off) |
| $t_u^{\text{prev}}$ | number of periods unit $u$ has been on or off prior to the first period of the planning horizon |

*Decision variables*

| | |
|---|---|
| $y_{ut}$ | 1 if unit $u$ is on in period $t$, 0 otherwise |
| $p_{ut}$ | production level of unit $u$, in period $t$ |

*Auxiliary variables*

| | |
|---|---|
| Auxiliary variables | Due to model structure, some of the auxiliary variables can be relaxed to the set $[0,1]$, as discussed later |
| $x_{ut}^{\text{on}}, x_{ut}^{\text{off}}$ | 1 if unit $u$ is started/switched off in period $t$, 0 otherwise |
| $s_{ut}^{\text{hot}}$ | 1 if unit $u$ has a hot start in period $t$, 0 otherwise |
| $s_{ut}^{\text{cold}}$ | 1 if unit $u$ has a cold start in period $t$,0 otherwise |
| $p_{ut}^{\max}$ | maximum production levels for unit $u$ in period $t$ (due to ramp constraints) |

*Production costs*

| | |
|---|---|
| $F(p_{ut})$ | fuel cost for unit $u$ in period $t$ |
| $S(x_{ut}^{\text{off}}, y_{ut})$ | start up cost for unit $u$ in period $t$ |
| $H_{ut}$ | shut down cost for unit $u$ in period $t$ |

The dramatic increase in the efficiency of mixed-integer programming (MIP) solvers has encouraged the thorough exploitation of their capabilities. Some research has already been directed towards defining of alternative, more efficient, mixed-integer linear programming (MILP) formulations of this problem (see e.g., [3]). Extensive surveys of different optimisation techniques and modelling issues are provided in [4–6].

This paper proposes a MIP formulation for quadratic optimisation of the UCP, and also presents a method based on a linear formulation. The method has proven to be effective at solving instances of a practically relevant size. Instead of considering a quadratic representation of the fuel cost, the linear model considers a piecewise linear approximation of the function and updates it in an iterative process, by including additional pieces. Function updating is based on the solutions obtained in the previous iteration.

The solution approach developed in this research was tested on several well-known test instances that were not known to have been solved to optimality. For each of them, the new approach iteratively converged to the optimal solution, even for the largest benchmark instances.

## 2. Problem variants

Different modelling alternatives that reflect different problem issues, such as fuel, multiarea and emission constraints have been published (e.g., [7–9]). Security constraints [10] and market related aspects [11] have been addressed more recently.

The decentralised management of production has also introduced new issues to the area [12] and in some markets the problem has now been reduced to single-unit optimisation. However, for several decentralised markets the traditional problem is still very much similar to that of the centralised markets [3]; the main difference is the objective function that, rather than minimising production costs, maximises total welfare. Therefore, the techniques that apply for centralised production management will also be effective at solving many decentralised market production problems.

This paper considers the centralised UCP model. The objective of the problem is to minimise total production costs over a given planning horizon. The total production cost is expressed as the sum of fuel costs (quadratic functions that depend on the production level of each unit) and start-up costs. Start-up costs are represented by constants that depend on the last period when the unit was operating. In addition to the uninterrupted operation of the unit (i.e., no start-up cost[1]), two constants are defined: one constant for hot start-up costs when the unit has been off for a number of periods smaller than or equal to a given value, and the other for cold start-up costs. The following constraints will be included in the formulation: system power balance, system reserve requirements, unit initial conditions, unit minimum up and down times, generation limits and ramp constraints. For a standard quadratic mathematical formulation refer to [13].

## 3. MILP formulations for the UCP

For many years, approaches to solving the UCP were mainly based on Lagrangian relaxation and (meta) heuristics. This was due to the non-existence of exact approaches capable of coping with the computational complexity of the problem using reasonable resources. However, the dramatic improvement of MIP solvers in recent years suggested that an effort should be applied to studying "good" mathematical formulations of the problem, so that it can be handled by relevant solvers.

The first requirement is the linearisation of the various nonlinearities in the problem; namely, minimum up and down time constraints, minimum and maximum power production constraints (for problems that consider ramps), and the objective function.

Several efforts have been made to improve and strengthen the formulation of the UCP; pioneering work can be found in [14]. This work considers three sets of binary variables that model the state of each unit, start-ups, and shut-downs. The quadratic fuel cost function is represented by a piecewise-linear cost function. Initial attempts to solve the problem with standard branch-and-bound (B&B) proved to be inefficient. As a result, an extended version of the algorithm that considers problem-specific characteristics in the branching process was proposed. Results are provided for problems with up to 16 units and 14 time periods.

A thorough discussion of model linearisation, considering a perfect electricity spot market and a single unit (self-scheduling), is provided in [15]. The quadratic cost function is approximated using a piecewise linear cost function with $L$ segments. Three extra sets of variables are required when compared to the model in [14]: 0/1

[1] Notice that even in this situation there is a fixed component in the quadratic cost function.