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Discrete Optimization

Scheduling with controllable release dates and processing times: Total completion time minimization ☆

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Abstract

The single machine scheduling problem with two types of controllable parameters, job processing times and release dates, is studied. It is assumed that the cost of compressing processing times and release dates from their initial values is a linear function of the compression amounts. The objective is to minimize the sum of the total completion time of the jobs and the total compression cost. For the problem with equal release date compression costs we construct a reduction to the assignment problem. We demonstrate that if in addition the jobs have equal processing time compression costs, then it can be solved in $O(n^2)$ time. The solution algorithm can be considered as a generalization of the algorithm that minimizes the makespan and total compression cost. The generalized version of the algorithm is also applicable to the problem with parallel machines and to a range of due-date scheduling problems with controllable processing times.

Keywords: Scheduling; Single machine; Parallel machines; Total completion time; Controllable processing times; Controllable release dates; Due-date scheduling

1. Introduction

This paper continues the study of the single machine problem with controllable processing times and release dates started in [3]. In both papers it is assumed that the job processing times and release dates may vary within certain limits. Compressing any of the two parameters incurs the compression cost that is represented by a linear function of the compression amounts. Such problems arise in manufacturing systems, supply chain scheduling, computing systems that support imprecise computations, computer networks, etc. (see [2,3,7,10,11] for the examples and references).

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In [3] we study the problem of minimizing the makespan together with the compression cost. We construct a reduction to the assignment problem for the case of equal release date compression costs and develop an $O(n^2)$ algorithm for the case of equal release date compression costs and equal processing time compression costs. For the bicriteria version of the latter problem with agreeable processing times we suggest an $O(n^2)$ algorithm that constructs the breakpoints of the efficient frontier.

The current paper focuses on the problem of minimizing the sum of the total completion time and the compression cost. We adapt the techniques developed in [3] and suggest a generalized algorithm that is applicable to the problem with single machine or parallel machines and to a range of problems with a common due date and controllable processing times.

The main problem studied in this paper can be formulated as follows. Each job from the set $N = \{1,\ldots,n\}$ has to be processed on a single machine without preemption. Each job $i \in N$ is characterized by the two parameters: release date r_i and processing time p_i each of which can vary within certain limits:

$$\underline{r}_i \leqslant r_i \leqslant \bar{r}_i,$$
$$\underline{p}_i \leqslant p_i \leqslant \bar{p}_i.$$

Compressing the release date of job i from its 'normal' (maximum) value $\bar{r}_i$ by the amount $x_i = \bar{r}_i - r_i$ incurs a cost $\alpha_i x_i$. Similarly, compressing the processing time by the amount $y_i = \bar{p}_i - p_i$ incurs a cost $\beta_i y_i$.

Job i is called *crashed*, if its processing time is compressed to its minimum value, i.e., if $p_i = \underline{p}_i$, and job i is called *uncrashed*, if it has the maximum processing time, i.e., if $p_i = \bar{p}_i$. Job i is called *partially crashed*, if $\underline{p}_i < p_i < \bar{p}_i$. We will say that the job is *early* if it starts before $\bar{r}_i$.

The problem with controllable release dates and controllable processing times is defined on the set of feasible schedules $S = \{(x, y, \pi) | 0 \leqslant x_i \leqslant \bar{x}_i,\ 0 \leqslant y_i \leqslant \bar{y}_i, i \in N\}$, where π is a permutation of the jobs from N. We assume that the jobs start at their earliest possible times with respect to their release dates $\bar{r}_i - x_i$ and permutation π. Given a schedule $s \in S$, the job completion times C_i, $i = 1,\ldots,n$, can easily be determined.

The objective of the problem considered in [3] is to minimize

$$F_{\max}(x, y, \pi) = C_{\max}(x, y, \pi) + K(x, y),$$

where $C_{\max}(x, y, \pi) = \max_{i \in N}\{C_i(x, y, \pi)\}$ is the makespan of the schedule and $K(x, y) = \sum_{i=1}^{n}(\alpha_i x_i + \beta_i y_i)$ is the compression cost.

The objective of the problem considered in this paper is to minimize

$$F_{\Sigma}(x, y, \pi) = \sum C_i(x, y, \pi) + K(x, y),$$

where $\sum C_i(x, y, \pi) = \sum_{i=1}^{n} C_i(x, y, \pi)$ is the total completion time.

We denote the two problems by $P_{\max}$ and P_{Σ}, respectively. Adapting the three-field classification scheme from [5], problems $P_{\max}$ and P_{Σ} are denoted by $1|\bar{r}_i - x_i, \bar{p}_i - y_i|F_{\max}$ and $1|\bar{r}_i - x_i, \bar{p}_i - y_i|F_{\Sigma}$.

Following the traditional scheduling research for problems with controllable release dates [1,6,8,9,12,15–17], we make the following assumptions:

$$\underline{r}_i = \underline{r}, \quad \bar{r}_i = \bar{r}, \quad i \in N \tag{1}$$

and

$$\bar{r} - \underline{r} \geqslant \sum_{i=1}^{n} \bar{p}_i. \tag{2}$$

Observe that if (1) does not hold, then both problems $P_{\max}$ and P_{Σ} are NP-hard (problem $P_{\max}$ is studied in [19,21]; problem P_{Σ} is equivalent to the NP-hard problem of minimizing total weighted earliness and tardiness with arbitrary due dates). On the other hand, if (2) does not hold, then as shown in [21], problem $P_{\max}$ is NP-hard and as shown in [15] problem P_{Σ} is equivalent to the well-known NP-hard problem [13,14] of minimizing total earliness and tardiness with restricted common due date. All the results hold even if processing times are fixed (non-controllable).

The paper is organized as follows. Problem P_{Σ} with $\alpha_i = \alpha$ and arbitrary β_i is studied in Section 2. Similar to problem $P_{\max}$, it is NP-hard if α_i are arbitrary (Section 2.1) and it is reducible to an assignment problem if α_i are equal (Section 2.2).