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Model Relaxations for the Fuel Cost Minimization of Steady-State Gas Pipeline Networks

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Abstract—Natural gas, driven by pressure, is transported through pipeline network systems. As the gas flows through the network, energy and pressure are lost due to both friction between the gas and the pipes' inner wall, and heat transfer between the gas and its environment. The lost energy of the gas is periodically restored at the compressor stations which are installed in the network. These compressor stations typically consume about 3–5% of the transported gas. This transportation cost is significant because the amount of gas being transported worldwide is huge. These facts make the problem of how to optimally operate the compressors driving the gas in a pipeline network important.

In this paper, we address the problem of minimizing the fuel cost incurred by the compressor stations driving the gas in a transmission network under steady-state assumptions. In particular, the decision variables include pressure drops at each node of the network, mass flow rate at each pipeline leg, and the number of units operating within each compressor station. We present a mathematical model of this problem and an in-depth study of the underlying mathematical structure of the compressor stations. Then, based on this study, we propose two model relaxations (one in the

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compressor domain and another in the fuel cost function) and derive a lower bounding scheme. We also present empirical evidence that shows the effectiveness of the lower bounding scheme. For the small problems, where we were able to find optimal solutions, the proposed lower bound yields a relative optimality gap of around 15–20%. For a larger, more complex instance, it was not possible to find optimal solutions, but we were able to compute lower and upper bounds, finding a large relative gap between the two. We show this wide gap is mainly due to the presence of nonconvexity in the set of feasible solutions, since the proposed relaxations do a very good job of approximating the problem within each individual compressor station.

We emphasize that this is, to the best of our knowledge, the first time such a procedure (lower bound) has been proposed in over 30 years of research in the natural gas pipeline area.

Keywords—Natural gas, Pipelines, Transmission networks, Compressor stations, Steady state, Lower bounds, Nonconvex objective.

1. INTRODUCTION

Natural gas, driven by pressure, is transported through a pipeline network system. As the gas flows through the network, pressure (and energy) is lost due to both friction between the gas and the pipe inner wall, and heat transfer between the gas and its environment. To overcome this loss of energy and keep the gas moving, compressor stations are installed in the network, which consume part of the transported gas resulting in a fuel consumption cost. Principal concerns with both designing and operating a gas pipeline network are maximizing throughput and minimizing fuel cost. Numerical simulations based on either steady-state or transient models of the networks have been used to attempt to provide solutions to these problems. The problem we address in this paper is minimizing fuel cost for steady-state gas pipeline networks.

As the gas industry has developed, gas pipeline networks have evolved over decades into very large and complex systems. A typical network today might consist of thousands of pipes, dozens of stations, and many other devices, such as valves and regulators. Inside each station, there can be several groups of compressor units of various vintages that were installed as the capacity of the system expanded. Such a network may transport thousands of MMCFD (1 MMCFD = 10^6 cubic feet per day) of gas, of which 3–5% is used by the compressor stations to move the gas. It is estimated [1] that the global optimization of operations can save at least 20% of the fuel consumed by the stations. Hence, the problem of minimizing fuel cost is of tremendous importance.

With the aid of today's powerful digital computers, numerical simulation of gas pipeline networks can be very accurate. This opens the door to the development of optimization algorithms. Over the years, many researchers have attempted this with varying degrees of success. The difficulties of such optimization problems come from several aspects. First, compressor stations are very sophisticated entities themselves. They might consist of a few dozen compressor units with different configurations and characteristics. Each unit could be turned on or off, and its behavior is nonlinear. Second, the set of constraints that define feasible operating conditions in the compressors along with the constraints in the pipes constitute a very complex system of nonlinear constraints. Surfing on such a manifold to attempt to find global optimal solutions demands an in-depth understanding of its structure. Finally, operations of the valves and regulators may introduce certain discontinuities to the problems as well.

The purpose of this paper is first to provide an in-depth study of the underlying mathematical structure of the compressor stations. Then, based on this study, we present a mathematical model of the fuel cost minimization problem, and derive a lower bounding scheme based on the following two model relaxations:

(i) relaxation of the fuel cost objective function; and
(ii) relaxation of the nonconvex nonlinear compressor domain.

Finally, we present empirical evidence that shows the quality of the proposed relaxations.

The results are promising. For the small instances, where we were able to find both optimal solutions for the original problem (upper bound), and for the relaxed problem (lower bound)