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Cost minimization for periodic maintenance policy of a system subject to slow degradation

D.H. Park[a,*], G.M. Jung[b], J.K. Yum[b]

[a]*Department of Statistics, Hallym University, 1 Okchon Dong, Chunchon, 200-702, South Korea*
[b]*Department of Statistics, Dongguk University, Pil-Dong Chung-Gu, Seoul, 100-715, South Korea*

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Abstract

This paper considers a repairable system which undergoes preventive maintenance (PM) periodically and is minimally repaired at each failure. Most preventive maintenance models assume that the system improves at each PM so that the hazard rate is reduced to that of a new system or to some specified level. In this paper, we consider the situation where each PM relieves stress temporarily and hence slows the rate of system degradation, while the hazard rate of the system remains monotonically increasing. The optimal number and period for the periodic PM that minimize the expected cost rate per unit time over an infinite time span are obtained. We also consider the case when the minimal repair cost varies with time. Explicit solutions for the optimal periodic PM are given for the Weibull distribution case.

Keywords: Degradation; Preventive maintenance; Minimal repair; Expected cost rate; Hazard rate; Restoration interval

1. Introduction

Preventive maintenance (PM) is the action taken on a system while it is still operating, which is carried out in order to keep the system at the desired level of operation. The optimal PM policy not only reduces the cost of maintaining a system in satisfactory conditions, but also improves the productivity of the system. The term "optimum" means "minimizing the expected cost rate per unit time over finite or infinite time span".

The PM policies are adapted to slow the degradation process of the system while the system is operating and to extend the system life. A number of PM policies have been proposed in the literature. These policies are typically to determine the optimum interval between PMs to minimize the average cost over a finite time span. Barlow and Hunter [1] consider a PM policy of periodic replacement with minimal repair at any intervening failures. Nakagawa [6] proposes optimum policies when the preventive maintenance is imperfect. Nguyen and Murthy [7] study two types of PM policies for a repairable system and assume that the life distribution of a system changes after each repair in such a way that its failure rate increases with the number of repairs carried out. Murthy and Nguyen [5] study the optimal age replacement policy with imperfect preventive maintenance. The preventive is imperfect in the sense that it can cause failure of a non-failed system. Canfield [3] discusses a periodic PM model for which the PM slows the degradation process of the system, while the hazard rate keeps monotone increase. Chun [4] considers determination of the optimal number of periodic preventive maintenance operations during the warranty period.

Most of preventive maintenance models assume that the hazard rate of a repairable system after each PM is restored to like new or to some specified level. However, for most repairable systems, the maintenance action is not necessarily the replacement of the whole system, but is used to slow the rate of system degradation. Hence the system may not be restored to as good as new immediately after the completion of maintenance action.

In this paper, we consider a periodic PM policy which is assumed to relieve stress temporarily after each PM and hence slow the rate of system degradation. The system is maintained preventively at periodic times kx and is replaced by a new system at the Nth PM, where $k = 1, 2, \ldots, N$. If the system fails between PMs, it undergoes only minimal repair and hence, the hazard rate remains undisturbed by any of these minimal repairs. The expression to compute the expected cost rate per unit time is derived. We also obtain the optimal period x and the optimal number N for the periodic PM, which minimize the expected cost rate per unit time for an infinite time span.

* Corresponding author.
E-mail address: dhpark@sun.hallym.ac.kr (D.H. Park).

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Nomenclature

T time to failure of a system
$f(t)$, $F(t)$ pdf, life distribution of T for which $F(t) = 0$, $t < 0$
$h(t)$ hazard rate without PM
$h_{pm}(t)$ hazard rate under PM
x period of PM
N number of PMs to be performed for the next replacement
C_{mr} cost of minimal repair at failure
C_{pm} cost of PM
C_{re} cost of replacement with $C_{re} \geq C_{pm}$
$C_{mr}(t)$ cost of minimal repair when the system fails at age t
$C(x,N)$, $C_1(x,N)$ expected cost rate per unit time

Section 2 describes the periodic PM model and its assumptions. In Section 3, we present the expressions for the expected cost rate for the periodic PM. In Section 4, we consider the problem of finding the optimal period and number for the periodic PM policies simultaneously. Section 5 deals with a periodic PM policy for a repairable system when minimal repair cost varies with time. Section 6 presents the explicit solutions for the optimal periodic PM policies when the failure time follows a Weibull distribution.

2. Model and assumptions

We consider a periodic PM model for which each PM slows the system degradation. For such a model, the hazard rate keeps monotonically increasing, although the rate of degradation is reduced after each PM. The followings are assumed:

1. The PM slows the rate of system degradation.
2. If the system fails between PMs, it undergoes only minimal repair.
3. The system is maintained preventively at periodic times kx, $k = 1, 2, \ldots, N$.
4. The system is replaced by a new system at the Nth PM.
5. The times to conduct PM, minimal repair and replacement are negligible.

3. Expected cost rate

Let $\bar{F}(t) = 1 - F(t)$. The following definitions are frequently referred to in this paper.

Definition 3.1. The hazard rate of a life distribution F is defined as

$$h(t) = \frac{f(t)}{\bar{F}(t)}$$

for t such that $\bar{F}(t) > 0$ if $f(t)$ exists.

Definition 3.2. A life distribution F is IFR(DFR) if $h(t)$ is nondecreasing (nonincreasing) in $t \geq 0$.

In this paper we consider a periodic PM model, which is proposed by Canfield [3]. For such a model, the hazard rate keeps monotonically increasing, although the rate of degradation is reduced after each PM. If the system fails between PMs, it undergoes only minimal repair and hence, the hazard rate remains undisturbed by any of these minimal repairs. For Canfield's [3] model, the system which responds to PM has an increasing hazard rate indicating that the system degrades with time. He assumes that each PM reduces operational stress to that existing τ time units previous to the PM intervention, where τ is a restoration interval and is less than or equal to the period of PM. Under this assumption, the hazard rate $h_{pm}(t)$ is given by

$$h_{pm}(t) = h_{pm}(kx) + h(t - k\tau) - h(k(x - \tau)) \quad (1)$$

for $k = 0, 1, 2, \ldots$, where $kx < t \leq (k+1)x$, $h_{pm}(0) = h(0)$ and x is the time interval between PM interventions. By substituting recursively, Eq. (1) can be rewritten as

$$h_{pm}(t) = \begin{cases} h(t), & \text{for } 0 \leq t \leq x \\ \displaystyle\sum_{i=1}^{k} \{h((i-1)(x-\tau) + x) & \text{for } kx < t \leq (k+1)x, \\ \quad -h(i(x-\tau))\} + h(t - k\tau), & k = 1, 2, 3, \ldots \end{cases} \quad (2)$$

Fig. 1 shows a typical plot of the hazard rate $h_{pm}(t)$ under Canfield's periodic PM where $h_{pm}(0) = h(0) = 0$. In Fig. 1, we consider the cases when $\tau < x$ and $\tau = x$.

The expected cost rate for running the periodic PM policy during $[0,Nx]$ can be obtained in the following manner:

Expected cost rate per unit time

$$= [(\text{expected cost of minimal repair in } [0, Nx)) + (\text{expexcted cost of PM in } [0, Nx)) + (\text{expected cost of replacement})]/Nx. \quad (3)$$

Each expected cost given in Eq. (3) is obtained as follows:

(i) Expected cost of minimal repair in $[0,Nx)$

$$= C_{mr}\left(\sum_{k=0}^{N-1} \int_{kx}^{(k+1)x} h_{pm}(t)\,dt\right),$$

where $h_{pm}(t)$ is given in Eq. (2). This is obtained by applying the results of Boland [2].
(ii) Expected cost of PM in $[0, Nx) = (N-1)C_{pm}$.
(iii) Expected cost of replacement $= C_{re}$.

Using (i), (ii) and (iii), the expected cost rate per unit time for running the periodic PM policy during $[0,Nx]$ is obtained