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An exact algorithm for cost minimization in series reliability systems with multiple component choices ☆

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Abstract

In this paper, we present an exact method for cost minimization problems in series reliability systems with multiple component choices. The problem can be modelled as a nonlinear integer programming problem with a nonseparable constraint function. The method is of a combined Lagrangian relaxation and linearization method. A Lagrangian bound is obtained by solving the dual of a separable subproblem. An alternative lower bound is derived by 0–1 linearization method. A special cut-and-partition scheme is proposed to reduce the duality gap, thus ensuring the convergence of the method. Computational results are reported to show the efficiency of the method.

Keywords: Reliability system; Cost minimization; Multiple component choice; Lagrangian bound; Branch-and-bound method

1. Introduction

Reliability optimization problems are often encountered in many industrial and engineering applications. One of the popular techniques in improving the reliability of a series system is to use parallel redundancy. Fig. 1 illustrates the structure of the series–parallel network.

The components in Fig. 1 may represent electronic parts in a section of circuits, coolers and filters in a lubrication system, valves in a pipeline (see, e.g., [1,21,24]) or subsystems of a complicated communication networks.

In this paper, we consider the cost minimization problem in series system with multiple component choices. The problem is to minimize the cost of a series system under a minimum overall reliability requirement. The problem can be modelled as the following nonlinear integer programming:

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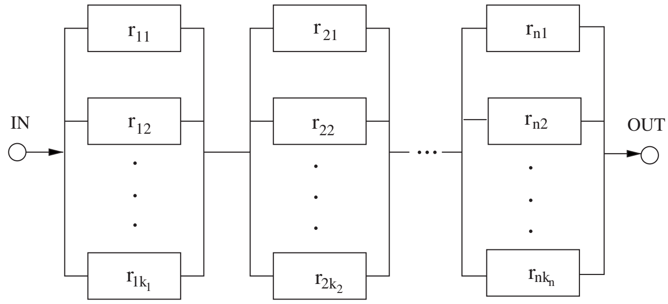

Fig. 1. Diagram of a series system with multiple component choices.

$$\begin{aligned}
(\mathrm{P}) \quad \min \quad & c(x)=\sum_{i=1}^{n}\sum_{j=1}^{k_i} c_{ij}(x_{ij}) \\
\text{s.t.} \quad & R(x)=\prod_{i=1}^{n}\left[1-\prod_{j=1}^{k_i}(1-r_{ij})^{x_{ij}}\right] \geqslant R_0, \\
& x \in X=\{x \mid 0 \leqslant x_{ij} \leqslant u_{ij}, x_{ij} \text{ integer}, i=1,\ldots,n, j=1,\ldots,k_i\},
\end{aligned}$$

where

- $r_{ij} \in (0,1)$: the reliability of the jth component (choice) in the ith subsystem in series;
- x_{ij}: the number of identical redundancy of the jth components in the ith subsystem;
- u_{ij}: the upper bound of the identical redundancy of the jth components in the ith subsystem;
- $c_{ij}(x_{ij})$: a convex and increasing function of x_{ij} represents the cost of having x_{ij} identical jth component in the ith subsystem;
- $1-\prod_{j=1}^{k_i}(1-r_{ij})^{x_{ij}}$: the reliability of the ith subsystem;
- $R(x)$: the overall system reliability when adopting redundancy assignment

  $$x=(x_{11},\ldots,x_{1k_1},\ldots,x_{n1},\ldots,x_{nk_n})^{\mathrm{T}};$$

- $R_0 \in (0,1)$: a given minimum reliability level.

Since r_{ij}'s can be different in the ith subsystem, the reliability function $R(x)$ is in general a nonseparable function. This makes it a great challenge to design efficient solution methods for (P). It is noticed that the well studied *simple* series–parallel system is a special case of problem (P) when $r_{i1}=\cdots=r_{ik_i}$ for $i=1,\ldots,n$ (see [5,19,24,25]).

Exact solution methods in reliability optimization are mainly for simple series–parallel system. For example, branch-and-bound methods and its combinations with dynamic programming methods, various partial enumeration techniques (see [11,14,17]) and cutting plane method [12]. Few implementable methods have been developed in the literature for reliability optimization problem with a nonseparable reliability function. Misra and Sharma [13] proposed a search algorithm to scan the entire feasible region of the optimal redundancy problem (see also [16]). Ng and Sancho [14] proposed a dynamic programming method combined with depth-first search technique. Chern and Jan [3] presented a two-phase method for solving the reliability problem. Sung and Cho [22,23] transformed the reliability problem into an equivalent binary integer problem and proposed a solution space reduction procedure to improve the algorithm. There are also heuristic methods that search for a near-optimal solution of reliability optimization problems. For example, genetic algorithms [4,6] and greedy methods [9] .

In this paper, we propose a new exact method for solving problem (P). This method is based on the Lagrangian dual search and linearization method. To overcome the nonseparability of the constraint, we first approximate $R(x)$ by a linear function. Lagrangian bounds of the approximation problem can be obtained by