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Pumping cost minimization in aquifers with regional flow and two zones of different transmissivities

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SUMMARY

In this paper we study minimization of pumping cost of a given total flow rate from any number and layout of wells. Building on previously published results, we have considered steady state flow in aquifers with two zones of different transmissivities, to which the method of images applies. Moreover, we have taken into account additional regional flow, which results in different hydraulic head values at the location of the wells, independent of their operation. We prove analytically that in this general case, pumping cost is minimized, when final differences between hydraulic head values at the locations of the wells, resulting from superposition of the regional flow and the operation of the system of the wells, are equal to the half of those, which are due to the regional flow only. Finally we present the analytical calculation procedure of the optimal distribution of the required total flow rate to the individual wells and we provide illustrative examples.

Introduction

Pumping cost minimization is one of the most common problems in groundwater resources management, e.g. Sidiropoulos and Tolikas (2004). Its complexity depends on the respective constraints, such as limits on well flow rates due to pump capacities or aquifer features and limits on hydraulic head drawdown in parts of the aquifer (e.g. Bayer et al., 2009). In other cases pumping cost is examined together with other cost items, such as amortization of well or pipe network construction cost, e.g. Wang and Zheng (1998), Cunha (1999). Water quality considerations, from seawater intrusion to nitrate pollution, may also enter the optimization process, e.g. Katsifarakis et al. (1999), Park and Aral (2004), Katsifarakis and Petala (2006), Minciardi et al. (2007). In many cases, pumping cost is the main item in aquifer restoration problems, e.g. Shieh and Peralta (2005), Matott et al. (2006), Papadopoulou et al. (2007).

Due to the importance of proper development of groundwater resources, the respective problems have been tackled over the years by many optimization methods, such as linear and non-linear programming (e.g. Bear, 1979; Rastogi, 1989; Theodossiou, 2004), genetic algorithms and other evolutionary techniques (e.g. Ouazar and Cheng, 2000; Mantoglou et al., 2004; Kalwij and Peralta, 2008; He et al., 2008), the outer approximation method (e.g. Spiliotopoulos et al., 2004), etc. Critical evaluations of different optimization methods have been presented by Mayer et al. (2002) and Fowler et al. (2008).

For steady flow in confined infinite aquifers, as well as in semi-infinite ones to which the method of images applies, the following proposition has been proved by Katsifarakis (2008): the cost to pump a given total flow rate Q_T from any number and layout of wells is minimized, when hydraulic head levels at all wells are equal to each other, as long as flow is due to that system of wells only.

In this paper we extend the aforementioned work to steady flows in aquifers with two zones of different transmissivities, to which the method of images applies. We prove that pumping cost is minimized when hydraulic head levels at all wells are equal to each other, as long as flow is due to that system of wells only. Moreover we outline the analytical calculation of the optimal distribution of Q_T to the individual wells and we provide an illustrative example.

Then we take into account regional flow, independent of the operation of the wells. We prove analytically that in this general case, pumping cost is minimized, when final differences between hydraulic head values at the locations of the wells, resulting from superposition of the regional flow and the operation of the system of the wells, are equal to the half of those, which are due to the

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Nomenclature

A	constant depending on energy cost	R	radius of influence of the system of wells (m)
b	aquifer width (m)	r_0	well radius (m)
C	variable pumping cost (m^4/s)	r_{IJ}	distance between wells I and J (m)
C_0	pumping cost (monetary units)	r_{Ij}	distance between well I and the image of well J
C_1	variable pumping cost in the presence of regional flow (m^4/s)	s_J	drawdown of hydraulic head at well J, due to the operation of the system of the wells (m)
h_J	distance between water level at well J and the predefined reference level (m)	T	aquifer transmissivity (m^2/s)
K	number of wells in zone 1	γ	transmissivity ratio (T_1/T_2)
N	total number of wells	δ	distance between the initial horizontal level of the hydraulic head and the predefined reference level (m)
P	critical point of C	δ_I	distance between the initial level of the hydraulic head at well I and the predefined reference level (m)
Q_J	flow rate of well J (l/s) or (m^3/s)		
Q_T	total flow rate pumped from the system of N wells (l/s)		

regional flow only. Finally we present the analytical calculation procedure of the optimal distribution of Q_T to the individual wells and we provide an illustrative example.

The form of the objective function

For any type of aquifer, pumping cost, namely the objective function of the minimization problem, can be defined as:

$$C_0 = A \cdot \sum_{I=1}^{N} Q_I \cdot h_I \tag{2.1}$$

where N is the number of wells, Q_I the flow rate of well I, h_I the distance between water level at well I and a predefined level (e.g. highest ground elevation), and A is a constant, depending on energy cost. Treating A as constant implies that pump efficiencies are considered as constants, too, and equal to each other.

When the initial hydraulic head level is horizontal, namely when flow is due to the system of wells only, Eq. (2.1) can be written as:

$$C_0 = A \cdot \sum_{I=1}^{N} Q_I \cdot (s_I + \delta) \tag{2.2}$$

where s_I is the drawdown of the hydraulic head at well I, which is due to the operation of the system of the wells, and δ is the distance between the initial horizontal level of the hydraulic head and the predefined reference level. Since δ is the same everywhere, the function C that should actually be minimized is:

$$C = \sum_{I=1}^{N} Q_I \cdot s_I \tag{2.3}$$

Well flow rates Q_I should not obtain negative values, since such values correspond to recharge wells. Moreover, they should fulfill the basic constraint of the problem, namely that their sum is equal to the required total flow rate Q_T:

$$\sum_{I=1}^{N} Q_I = Q_T \tag{2.4}$$

Aquifers with two zones of different transmissivities

Let us consider an infinite aquifer with two zones of different transmissivities. As shown in Fig. 1, interface between them, which coincides with the y-axis, is rectilinear. Moreover, wells 1 to K are in zone 1, while the rest of them ($K+1$ to N) lie in zone 2. Real wells are denoted with capital letters, while their images with lower case ones.

Hydraulic head level drawdown can be calculated analytically in this case, using the method of images (e.g. Bear, 1979), which applies to fields with one (or more under certain conditions) straight line boundaries. The basic idea of the method is that a boundary can be "removed" by adding a number of fictitious wells, symmetrical of the real ones with respect to it. The relationship between the flow rate of each fictitious (or image) well and that of the respective real one, depends on the boundary condition along the "removed" boundary and guarantees that it is observed.

In our case the configuration of the imaginary wells depends on the zone, too. Then, s_I for the wells 1 to K, which lie in zone 1, is given as:

$$\begin{aligned} s_I &= -\frac{1}{2\pi T_1}\sum_{J=1}^{K} Q_J \cdot \left(\ln\frac{r_{IJ}}{R} + \frac{T_1 - T_2}{T_1 + T_2}\cdot \ln\frac{r_{Ij}}{R}\right) \\ &\quad - \frac{1}{\pi(T_1+T_2)}\sum_{J=K+1}^{N} Q_J \cdot \ln\frac{r_{IJ}}{R} \Rightarrow s_I \\ &= -\frac{1}{2\pi T_1}\sum_{J=1}^{K} Q_J \cdot \ln\frac{r_{IJ}}{R} - \frac{T_1 - T_2}{2\pi T_1(T_1+T_2)}\cdot\sum_{J=1}^{K} Q_J \cdot \ln\frac{r_{Ij}}{R} \\ &\quad - \frac{1}{\pi\cdot(T_1+T_2)}\sum_{J=K+1}^{N} Q_J \cdot \ln\frac{r_{IJ}}{R} \end{aligned} \tag{3.1}$$

while for wells, $K+1$ to N, which lie in zone 2, s_I is given as:

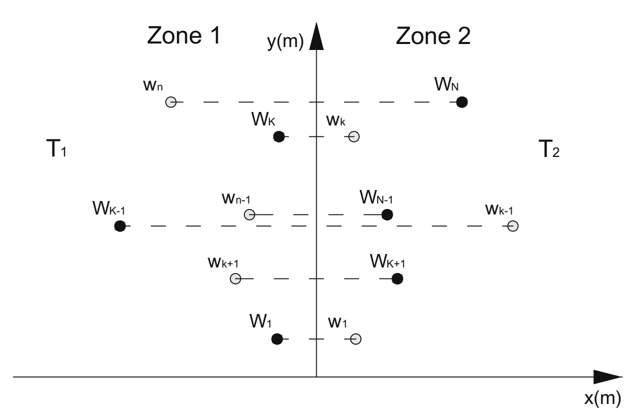

Fig. 1. Typical system of wells and their images in an aquifer with two zones of different transmissivities (real and image wells are denoted by capital and lower case letters, respectively).