Expert Systems with Applications 39 (2012) 869–880

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Uncertain induced aggregation operators and its application in tourism management

José M. Merigó [a,*], Anna M. Gil-Lafuente [a], Onofre Martorell [b]

[a] Department of Business Administration, University of Barcelona, Av. Diagonal 690, 08034 Barcelona, Spain
[b] Department of Business Administration, University of Balearic Islands, Palma de Mallorca, Spain

ARTICLE INFO

Keywords:
Interval numbers
Weighted average
OWA operator
Aggregation operators
Tourism management
Multi-person decision-making

ABSTRACT

We develop a new decision making approach for dealing with uncertain information and apply it in tourism management. We use a new aggregation operator that uses the uncertain weighted average (UWA) and the uncertain induced ordered weighted averaging (UIOWA) operator in the same formulation. We call it the uncertain induced ordered weighted averaging – weighted averaging (UIOWAWA) operator. We study some of the main advantages and properties of the new aggregation such as the uncertain arithmetic UIOWA (UA-UIOWA) and the uncertain arithmetic UWA (UAUWA). We study its applicability in a multi-person decision making problem concerning the selection of holiday trips. We see that depending on the particular type of UIOWAWA operator used, the results may lead to different decisions.

1. Introduction

The weighted average (WA) is one of the most common aggregation operators found in the literature. It can be used in a wide range of problems including statistics, economics and engineering. Another interesting aggregation operator is the ordered weighted averaging (OWA) operator (Yager, 1988). The OWA operator provides a parameterized family of aggregation operators that range from the maximum to the minimum. For further reading on the OWA operator and some of its applications, refer to Ahn (2009), Beliakov, Pradera, and Calvo (2007), Chang and Wen (2010), Cheng, Wang, and Wu (2009), Kacprzyk and Zadrozny (2009), Liu, Cheng, Chen, and Chen (2010), Merigó (2010a, 2010b), Merigó, Casanovas, and Martínez (2010), Merigó and Gil-Lafuente (2008, 2010, 2011a), Wei (2010a), Xu (2009, 2010), Xu and Da (2003), Xu and Yager (2010), Yager (1993, 1998, 2009), and Yager and Kacprzyk (1997), Zhao, Xu, Ni, and Liu (2010), Zeng & Su (2011), Zhou and Chen (2010). An interesting generalization of the OWA operator is the induced OWA (IOWA) operator (Yager & Filev, 1999). Its main advantage is that it deals with complex reordering processes in the aggregation by using order inducing variables. Since its introduction it has been studied by a lot of authors. For example, Merigó and Gil-Lafuente (2009) developed a generalization by using generalized and quasi-arithmetic means. Chiclana, Herrera-Viedma, Herrera, and Alonso (2004) and Xu and Da (2003) introduced a geometric version and applied it in group decision making. Wu, Li, Li, and Duan (2009) presented a continuous geometric version. Merigó and Casanovas (2009) applied it in a decision making problem with Dempster–Shafer belief structure. Chen and Chen (2003) and Merigó and Casanovas (2010) studied the use of fuzzy numbers in the IOWA operator. For further reading, see Merigó (2011), Merigó, Gil-Lafuente, and Gil-Aluja (2011a, 2011b), Tan and Chen (2010), Wei (2010b), Wei, Zhao, and Lin (2010) and Yager (2003).

Usually, when using these approaches it is considered that the available information are exact numbers. However, this may not be the real situation found in the specific problem considered. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with exact numbers. Therefore, it is necessary to use another approach that is able to assess the uncertainty such as the use of interval numbers. By using interval numbers we can consider a wide range of possible results between the maximum and the minimum. Note that in the literature, there are a lot of studies dealing with uncertain information represented in the form of interval numbers (Merigó, López-Jurado, Gracia, & Casanovas, 2009; Merigó & Wei, 2011; Wei, 2009; Xu & Da, 2002; Xu & Da, 2003; Xu & Yager, 2010).

Recently, some authors have tried to unify the WA and the OWA in the same formulation. It is worth noting the work developed by Torra (1997) with the introduction of the weighted OWA (WOWA) operator and the work of Xu and Da (2003) concerning the hybrid averaging (HA) operator. Both models arrived to a partial unification between the OWA and the WA because both concepts were included in the formulation as particular cases. However, as it has been studied by Merigó (2008), these models seem to be a partial unification but not a real one because they can unify them but they

* Corresponding author. Tel.: +34 93 402 19 62; fax: +34 93 403 98 82.
E-mail addresses: jmerigo@ub.edu (J.M. Merigó), amgil@ub.edu (A.M. Gil-Lafuente), onofre.martorell@uib.es (O. Martorell).

0957-4174/$ - see front matter
doi:10.1016/j.eswa.2011.07.085

cannot consider how relevant these concepts are in the specific problem considered. For example, in some problems we may prefer to give more importance to the OWA operator because we believe that it is more relevant and vice versa. This problem is solved with the ordered weighted averaging – weighted averaging (OWAWA) operator (Merigó, 2008, 2010b).

In this paper, we present a new approach to unify the IOWA operator with the WA when the available information is uncertain and can be assessed with interval numbers. We call it the uncertain induced ordered weighted averaging – weighted averaging (UIOWAWA) operator. The main advantage of this approach is that it unifies the OWA and the WA taking into account the degree of importance that each concept has in the formulation and considering that the information is given with interval numbers. Thus, we are able to consider situations where we give more or less importance to the UIOWA and the UWA depending on our interests and the problem analyzed. Furthermore, by using the UIOWAWA, we are able to use a complex reordering process in the OWA operator in order to represent complex attitudinal characters. We also study different properties of the UIOWAWA operator and further generalizations such as the mixture UIOWAWA (MUIOWAWA) operator and the infinitary UIOWAWA operator. Moreover, we discuss several particular cases including the UWA, the UIOWA, the uncertain arithmetic UIOWA (UA-UIOWA) and the uncertain arithmetic UWA (UAUWA).

We also analyze the applicability of the new approach and we see that it is possible to develop an astonishingly wide range of applications. For example, we can apply it in a lot of problems regarding statistics, economics, engineering and decision theory. In this paper, we focus on a decision making problem concerning tourism management. We develop a multi-person decision making problem where a decision maker wants to select an optimal holiday trip. The main advantage of the UIOWAWA in these problems is that it is possible to consider the subjective probability (or the degree of importance) and the attitudinal character of the decision maker at the same time.

This paper is organized as follows. In Section 2 we revise some basic concepts and in Section 3 we suggest a new method for dealing with uncertain weights in the aggregation process. In Section 4 we present the new aggregation operator. Section 5 analyzes the applicability of the UIOWAWA operator including an application in tourism management and in Section 6 we summarize the main results of the paper.

2. Preliminaries

In this Section we briefly review the interval numbers, the UWA, the UOWA operator, the UIOWA operator and the OWAWA operator.

2.1. The interval numbers

The interval numbers (Bachs, Merigó, López-Jurado, & Gracia-Ramos, 2008; Moore, 1966) are a very useful and simple technique for representing uncertainty. They have been used in a wide range of applications and can be defined as follows.

Definition 1. Let $a = [a_1, a_2] = \{x | a_1 \leqslant x \leqslant a_2\}$, then, a is called an interval number. Note that a is a real number if $a_1 = a_2$.

The interval numbers can be expressed in different forms. For example, assume a 4-tuple $[a_1, a_2, a_3, a_4]$, that is to say, a quadruplet, and let a_1 and a_4 represent the minimum and the maximum of the interval number, respectively, and a_2 and a_3 represent the interval that it is most possible to occur. Note that $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$. If $a_1 = a_2 = a_3 = a_4$, then the interval number is an exact number. If $a_2 = a_3$, it is a triplet, and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we review some basic operations. Let A and B be two triplets, where $A = [a_1, a_2, a_3]$ and $B = [b_1, b_2, b_3]$.

1. $A + B = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$.
2. $A - B = [a_1 - b_3, a_2 - b_2, a_3 - b_1]$.
3. $A \times k = [k \times a_1, k \times a_2, k \times a_3]$, for $k > 0$.
4. $A \times B = [\min(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3), a_2 \times b_2, \max(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3)]$, for R.
5. $A \div B = [\min(a_1 \div b_1, a_1 \div b_3, a_3 \div b_1, a_3 \div b_3), a_2 \div b_2, \max(a_1 \div b_1, a_1 \div b_3, a_3 \div b_1, a_3 \div b_3)]$, for R.

Sometimes, the ranking of the intervals is difficult because it is not clear which interval number is higher, so we must establish an additional criterion for ranking the interval numbers. For simplicity, we follow the following method throughout the paper. For 2-tuples, calculate the arithmetic mean of the interval, with $(a_1 + a_2)/2$. For 3-tuples and above, calculate a weighted average that yields more importance to the central values. That is, for 3-tuples, $(a_1 + 4a_2 + a_3)/6$. For 4-tuples, we calculate: $(a_1 + 4a_2 + 4a_3 + a_4)/10$, and so on. In the case of a tie between the intervals, we select the interval with the lowest difference, i.e., $(a_2 - a_1)$. For 3-tuples and above odd-tuples, we select the interval with the highest central value. Note that for 4-tuples and above even-tuples, we must calculate the average of the central values following the initial criteria.

The main advantage of this method is that we can reduce the interval number into a representative and exact number of the interval. To understand the usefulness of this method, we present a simple example.

Example 1. Assume we want to rank the following interval numbers: $A = (38, 47, 57)$, $B = (39, 45, 55)$ and $C = (42, 46, 51)$. Initially, it is not clear which is higher. Obviously, we can use a wide range of methods depending on the importance we want to give to each tuple of the interval. We assume in this paper a ranking based on $(a_1 + 4a_2 + a_3)/6$. Thus, we assume that the central value is more important than the extreme values. We convert the triplet to exact numbers. Thus:

$$A = (38 + 4 \times 47 + 57)/4 = 47.16.$$

$$B = (39 + 4 \times 45 + 55)/4 = 45.66.$$

$$C = (42 + 4 \times 46 + 51)/4 = 46.16.$$

With these results, we can reorder the interval numbers such that $A > C > B$.

Note that other operations and ranking methods could be studied (Bachs et al., 2008; Moore, 1966) but in this paper we focus on those discussed above.

2.2. The uncertain weighted average

The uncertain weighted average (UWA) is an extension of the weighted average for situations in which the available information is uncertain and can be assessed using interval numbers. It can be defined as follows.

Definition 1. Let Ω be the set of interval numbers. An UWA operator of dimension n is a mapping UWA: $\Omega^n \to \Omega$ that has an associated weighting vector W of dimension n with $\tilde{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \tilde{w}_i = 1$ such that:

$$\mathrm{UWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{i=1}^{n} \tilde{w}_i \tilde{a}_i, \tag{1}$$

where $\tilde{a}_i$ is an interval number.