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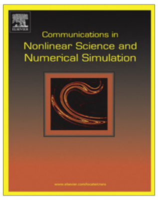

Ghost stochastic resonance induced by a power-law distributed noise in the FitzHugh–Nagumo neuron model

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ABSTRACT

We numerically investigate the ghost stochastic resonance phenomenon induced by a power-law distributed noise in the neuron FitzHugh–Nagumo model. The input noise considered is produced by a Langevin process including both multiplicative and additive Gaussian noise sources. In this process, the power-law decay exponent of the resulting noise distribution is governed by the off-set of the multiplicative noise, thus allowing us to probe both regimes of Gaussian and strongly non-Gaussian noises. Ghost stochastic resonance, i.e., stochastic resonance in a missing fundamental harmonic, occurs in this model. Deviations from the Gaussianity of the input noise are shown to reduce both the additive noise intensity corresponding to the optimal identification of the missing fundamental as well as the number of firing events at the ghost stochastic resonance condition.

1. Introduction

The influence of noise on the time evolution of dynamical systems is quite diverse. Besides its standard effect of producing disorder and, therefore, compromising the identification of a regular input signal, it can have the opposite effect of improving the system's response to an external stimulus under appropriated conditions. In particular, the study of neural systems under the influence of a noise source is fundamental to fully understand its dynamics. For example, neurons in vitro fire with considerable regularity in response to a constant stimulus while neurons in vivo exhibit a much larger irregularity in response to the same stimulus. A number of possible sources for neuronal noise in vivo has been proposed, including an intrinsic channel [1], Johnson electrical [2] and network [3] noises. Among the many positive roles played by noise in dynamical systems, the stochastic resonance (SR) phenomenon is one of the most intriguing. SR is the optimal detection of a sinusoidal sub-threshold signal achieved at a characteristic noise level. Along the years, SR has contributed to change the traditional concept of noise as a disturbing agent [4–15].

Recently, SR has been used to explain a phenomenon that happens in the human ear known as the "missing fundamental illusion". In this phenomenology, the human ear can perceive tones that are not present in the characteristic function of pressed notes [16–19]. This phenomenon has been studied in a model of neurons and compared with the general case of unharmonious tones used in the experiments of Schouten [20] where the complex signal was constructed adding pure high-order harmonics of a fundamental frequency. Ghost resonance produced by noise was observed with good agreement

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with experiments. The missing frequency resonance phenomenon promoted by noise has been termed as *ghost stochastic resonance* (GSR) and it has been detected in different systems like lasers [21], electronic circuits [22] and chaotic systems [23].

Usually, GSR frequency is investigated when a given dynamical system is driven by a white Gaussian noise source. However, non-Gaussian noises are quite frequent in biological systems. For example, some experimental results in sensory systems, particularly for one kind of crayfish, as well as in rat skin experiments, offer strong indications that the noise source in these systems could be non-Gaussian [24]. Experimental studies in rat sensory neurons have demonstrated that, under certain circumstances, colored noise can be better than white Gaussian noise for enhancing the neuron response to a weak signal [25]. Although several studies have addressed the question of the influence of the noise statistical properties on the stochastic identification of weak harmonic signals [26], there still lacks a better understanding of how the noise characteristics can affect the missing fundamental illusion in sensory neuron models.

In the present work, we investigate how the noise deviation from Gaussianity influences the GSR characteristics exhibited by a nonlinear sub-threshold dynamical system. As a prototype model, we will consider the FitzHugh–Nagumo (FHN) neural model driven by superposed harmonics of a missing fundamental and a power-law distributed noise source resulting from a Langevin process including both additive and multiplicative Gaussian noises. By controlling the model parameters, one can tune the degree of deviation from Gaussianity and explore its influence on the characteristic noise intensity leading to an optimal identification of the missing fundamental.

This work is organized as follows: In Section 2, we review some basic aspects of stochastic and ghost resonances. In Section 3, we describe the main features of the FitzHugh–Nagumo neural model. The Langevin process leading to the production of a power-law distributed noise is detailed in Section 4. Section 5 is devoted to the numerical methodology we used and to the main results characterizing the GSR phenomenon in the modeled system. Finally, we summarize and draw our main conclusions in Section 6.

2. Stochastic and ghost resonances

When a nonlinear dynamical system is driven by a sub-threshold periodical signal, the superposition of an input noise can make the output signal to bring information regarding the external sub-threshold input. Usually there is an optimal intensity of the superposed noise that leads to the highest resolution of the sub-threshold periodic signal. This effect is known as SR and it has been studied in several different physical scenarios such as lasers, chemical reactions, and chaotic systems [27–29].

The ghost resonance is a variant of the SR phenomenon in which the periodic stimulus is a superposition of higher harmonics, equally spaced in frequency, of a fundamental tone [16–23]. When the maximum of this complex signal is sub-threshold, a SR condition can be reached in the presence of noise. However, the main SR is not in any of the frequencies contained in the periodic stimulus but rather in the missing fundamental tone. This phenomenon is referred as the missing fundamental illusion, or in this case, GSR because the perceived tone corresponds to the fundamental frequency for which there is no actual source. It only appears in the output signal due to the presence of noise. This phenomenon has been shown to be directly related to pitch perception of complex sound waves [17]. Within this context, a relevant question refers to the shift in the pitch perception when the frequencies of the harmonic tones are rigidly displaced, which makes them no longer higher harmonics of a fundamental tone. The external complex stimulus is usually considered as a superposition of sinusoidal functions in the form

$$F(t) = A\{sin(2\pi f_1 t) + sin(2\pi f_2 t) + \cdots + sin(2\pi f_n t)\} \tag{1}$$

where $f_1 = kf_0 + \Delta f; f_2 = (k+1)f_0 + \Delta f; \ldots; f_n = (k+n-1)f_0 + \Delta f$. Here, Δf is a frequency shift from a perfect harmonic series, f_0 is the fundamental tone, and A is the amplitude of the signal components. A SR is observed in frequencies given by Ref. [18,19]

$$f_r = f_0 + \Delta f/[k + (n-1)/2], \tag{2}$$

where $n = 1, 2, 3, \ldots$ and $k > 1$. The above equation actually corresponds to the expected frequency at which the highest peaks of the complex signal occur. Such prediction has been probed in several physical systems such as the neuron model [18], semiconductor lasers [22], chaotic systems [23] and electronic circuits [21,23]. It has also been well reproduced in experiments of pitch perception [20]. A recent review on GSR and its different manifestations can be found in Ref. [16].

3. The FitzHugh–Nagumo neuron model

The FitzHugh–Nagumo neuron (FHN) model is a representative example of a bistable excitable system that occurs in a wide range of applications ranging from kinetics of chemical reactions to biological processes. Different aspects of the dynamics of this model and similar excitable ones in the presence of noise have been discussed from different points of view [30–34]. The equation of motion for the FHN neuron is given by

$$\begin{cases} \epsilon \frac{dv}{dt} = v(v-a)(1-v) - \omega + F(t) + \nu(t) \\ \frac{d\omega}{dt} = v - \omega - b \end{cases} \tag{3}$$