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# Diagnostics of stochastic resonance using Poincaré recurrence time distribution

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ABSTRACT

In the paper we calculate the distribution density of Poincaré recurrence times for a one-dimensional nonhyperbolic cubic map subjected to white noise and a harmonic signal. It is established that for small vicinities of recurrence the distribution density is not described by an exponential law and is periodically modulated with the external signal frequency. It is shown that the Fourier spectrum of the distribution density exhibits a well-distinguishable peak at the external signal frequency. The peak amplitude achieves its maximal value in the regime of stochastic resonance (SR), that can be used for detecting the SR effect.



The Poincaré recurrence time is one of the fundamentally important characteristics of temporal dynamics of chaotic systems. It has been proven by Poincaré in 1912 (the proof is presented in Ref. [1]) that for stable according to Poisson systems with a defined probability measure, a phase trajectory starting from an initial point $\vec{x}_0(t_0)$ will return infinitely often to its $\varepsilon$-vicinity. The first Poincaré recurrence times in a chaotic system represents a random sequence $\tau_r(k) = t_{k+1} - t_k$, where $t_k$ are the moments of sequential recurrences of the trajectory to the $\varepsilon$-vicinity. Thus, the probability distribution density $p(\tau_r)$ can be used to define the moments $\langle\tau_r^n\rangle$:

$$\langle\tau_r^n\rangle = \int_0^\infty \tau_r^n p(\tau_r) d\tau. \tag{1}$$

Nowadays the rigorous theory for Poincaré recurrences has been developed well enough [1]. The interrelation has been established between the average recurrence time to an $\varepsilon$-vicinity of certain point $\vec{x}_0$ with probability $P(\varepsilon, \vec{x}_0)$ (Kac's theorem [2]), and an expression for the probability measure has been obtained [3–5]:

$$p(\tau_r) = \frac{1}{\langle\tau_r\rangle} \exp\left(-\frac{\tau_r}{\langle\tau_r\rangle}\right), \quad \tau_r \geqslant \tau_r^*. \tag{2}$$

The distribution law (2) has been proven for ergodic systems that belong to a class of hyperbolic systems [1,6,7]. For certain conditions this expression is valid for chaotic systems with normal dynamics [1]. Numerous experiments testify that the law (2) can be approximately applied to nonhyperbolic systems [8,9]. It has to be noted that the distribution (2) is valid only when $\varepsilon$ is sufficiently small. Strictly speaking, the law (2) holds only in the limit $\varepsilon \to 0$ which is experimentally substantiated in Ref. [9]. As established in Refs. [9,10], if $\varepsilon$ is small but finite ($0 < \varepsilon < 1$), the distribution density differs from (2) but reflects some important dynamics peculiarities of the system under study. This property of $p(\tau_r)$ for finite and small $\varepsilon$ will be used in our paper.

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The objective of the paper is to substantiate a new method of diagnosing the stochastic resonance (SR) effect by analysing the distribution density $p(\tau_r)$ for the first Poincaré recurrence times to a small but finite vicinity $\varepsilon$ of an arbitrary point on an attractor of a bistable system. The SR phenomenon was originally published in Ref. [11,12] where the overdamped Kramers oscillator has been used as an example:

$$\dot{x} = x - x^3 + A\cos\Omega t + \sqrt{2D}\xi(t), \tag{3}$$

where $A$ and $\Omega$ are the amplitude and frequency of an external force, $D$ is the intensity of $\delta$-correlated noise $\xi(t)$.

It has been established and repeatedly verified in experiments ([13] and references therein) that in the regime of noise-induced switchings the intensity of a periodic component of the output signal $x(t)$ spectrum reaches its maximum at an optimal noise level $D = D^*$. It has been shown that for the SR regime the Kramers switching frequency [14,15] at the noise level $D^*$ coincides with the external signal frequency $\Omega$ in (3). The dependence of the signal-to-noise ratio (SNR) on the noise intensity $D$ resembles a resonant curve with maximum at $D = D^*$, that is why this effect is called as stochastic resonance.

We consider one of the simplest systems that can demonstrate SR both in the classical case of noise-induced switchings and in a noiseless case when a control parameter is varied (in the presence of crisis of two symmetrical attractors [16]):

$$x_{n+1} = (ax_n - x_n^3)\exp\left(-\frac{x_n^2}{b}\right) + A\sin(\Omega n) + \sqrt{2D}\xi(n). \tag{4}$$

The system (4) is a one-dimensional cubic map subjected to a weak ($A \ll 1$) periodic signal and a $\delta$-correlated noise source $\xi(n)$ with intensity $D$.

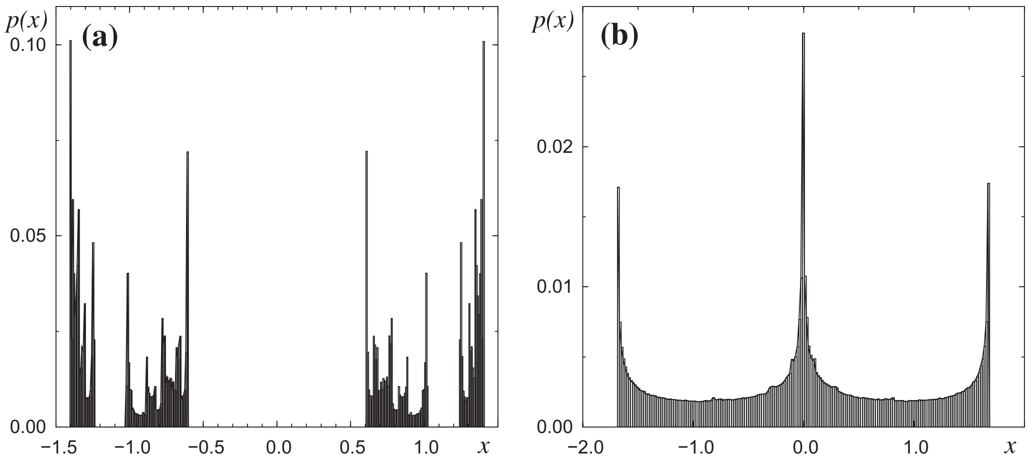


**Fig. 1.** Probability distribution density for (4) with $A = D = 0,\ b = 10$ and $a = 2.5$ (a), $a = 2.84$ (b).

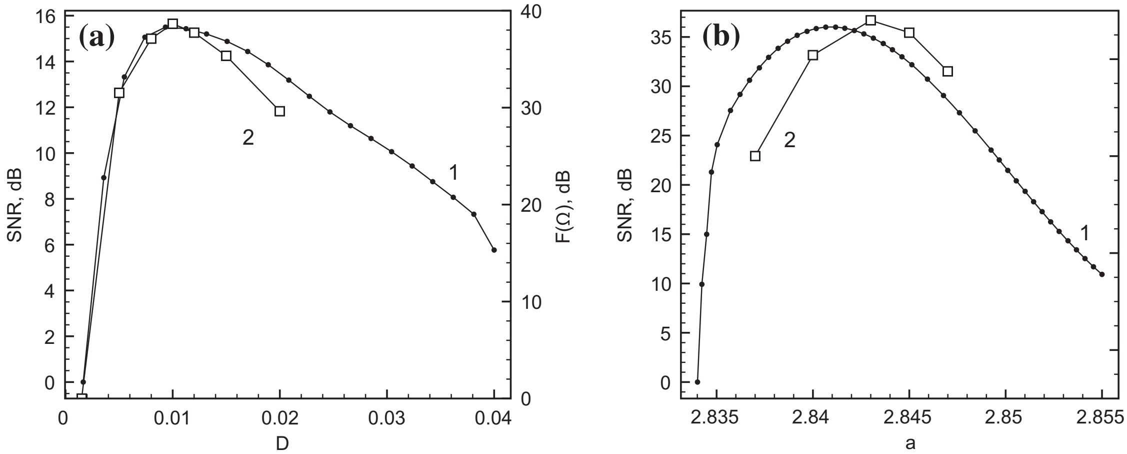


**Fig. 2.** SNR (curves 1) and the maximal value of normalized spectrum function at the external frequency $F(\Omega)$ (curves 2) (a) as functions of the noise intensity $D$ for $a = 2.5,\ A = 0.05,\ \Omega = 0.1$, and (b) of the control parameter $a$ for $D = 0,\ A = 0.005,\ \Omega = 0.1$ [16].