Applied Mathematical Modelling 38 (2014) 2915–2921

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Stochastic resonance in a linear static system driven by correlated multiplicative and additive noises

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ARTICLE INFO

Article history:
Received 23 October 2011
Received in revised form 22 September 2013
Accepted 22 November 2013
Available online 4 December 2013

Keywords:
Linear static system
Correlated noises
Stochastic resonance

ABSTRACT

In this work, we investigate the signal transmission in a linear static system driven by correlated multiplicative and additive noises. When the input signal is periodic, we depict the stochastic resonance (SR) phenomenon by employing the signal-to-noise ratio (SNR) theory; while the input signal is aperiodic, we describe the SR phenomenon by using the input–output cross correlation theory. And the exact analytic expressions of the output SNR and the normalized time averaged cross covariance between input and output are obtained. The results show: under the condition of negative correlated noises, SR arises; while with positive correlated or uncorrelated noises, there is no SR. This result may extend the SR theory to a common linear static system.

1. Introduction

Stochastic resonance (SR) is observed in many nonlinear systems that are driven by a signal and noise simultaneously, and it was proposed firstly as a model to explain the periodic appearance of the ice ages in 1981. SR shows that an increase of the input noise level may results in an increase of the signal-to-noise ratio (SNR) at the output of the nonlinear system. SR gives examples that noise can play a counterintuitive constructive role, also due to the ubiquitous noise in neural systems, chemistry systems, signal processing and other fields, so investigators in various fields pay attention to SR recently, and people try their best to discover the conditions under which SR will arise. At the primary stage of studying SR, people found that nonlinearity is a necessary ingredient for the appearance of SR [1,2]. However, with the development of the study on SR, investigators observed SR in some linear system, for example, Fulinski [3] found that SR can also occur in linear systems driven by non-Markovian dichotomic noise; Calisto et al. [4] observed SR in a linear system driven by Ornstein–Uhlenbeck process; Cao et al. [5] investigated SR in a periodically driven linear system with multiplicative and periodically modulated additive white noises. We should note from these examples, both non-Markovian dichotomic noise and Ornstein–Uhlenbeck process must have non-zero correlation time. So these instances show that the correlation in noises is important in the appearance of SR in linear system. Moreover, correlated noises can cause new and interesting phenomena in nonlinear system, for instance, researchers found in a bistable system that correlated noises can cause SR or doubly SR phenomena [6–8], doubly SR means double peak will appear in the curve of the output SNR; Also investigations [9,10] indicate that cross-correlation noise can induce non-equilibrium phase transition and reentrance phenomenon in nonlinear system. So it is significative to study the role of correlated noises in signal transmission.

On the other hand, most investigations about SR have focused on dynamic systems. With the developments of the theories and methods on SR, SR has been extended far and wide. For example, Chapeau-Blondeau and his collaborators [11–14] have extended the notion of SR from dynamic system to static (or memory less) nonlinear system, they observed SR in a

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http://dx.doi.org/10.1016/j.apm.2013.11.013

two-threshold nonlinearity system, also in a diode like nonlinearity system, and so on. However, as far as the author knows, most investigations on SR in static systems focus on nonlinear sensors. However, linear transmission systems are popular and important in practice. This provides motivation to investigate the signal transmission and SR phenomenon in a linear static system driven by correlated noises, Dong [15,16] tried to discover the condition under which SR will appear in static linear system driven by correlated noises, however, in [15,16] the author has not observed SR phenomenon in linear static system. In this paper, the author obtained the condition under which SR will arise in linear static system.

We consider the signal transmission in a linear static system driven by correlated noises in this paper. In Section 2, we give the model; In Section 3, when the input signal is periodic we depict the SR phenomenon by employing the SNR theory; and in Section 4, with the input signal is aperiodic, we discuss the SR by using the normalized time averaged cross covariance between input and output signal; Section 5 is the conclusion.

2. The linear transmission model

We consider the following linear static system realizing the input–output transformation [11]:

$$y(t) = h[s(t)(1+\xi(t)) + \eta(t)], \tag{1}$$

$$h[u] = cu, \quad c \neq 0 \text{ is a constant}, \tag{2}$$

where t is the time variable, $s(t)$ denotes the input signal, it can be a periodic signal or an aperiodic signal. And $y(t)$ is the steady-state response of the system (1) and (2), here steady-state response means that we consider $y(t)$ when the inputs signal $s(t)$ have been applied since $t \to -\infty$ [11]. $\xi(t)$ is the multiplicative noise and $\eta(t)$ is the additive noise, and we suppose they are aroused by the system (1) and (2). In this paper, we let $f_\xi(x)$ and $f_\eta(x)$ are the probability density functions of $\xi(t)$ and $\eta(t)\dot{y}$ respectively, moreover, $\xi(t)$ and $\eta(t)$ are correlative. The noise terms $\xi(t)$ and $\eta(t)$ are characterized by their mean and variance as:

$$E\xi(t) = \int_{-\infty}^{+\infty} x f_\xi(x)dx = 0, \quad E\eta(t) = \int_{-\infty}^{+\infty} x f_\eta(x)dx = 0,$$

$$E\xi(t)^2 = \int_{-\infty}^{+\infty} x^2 f_\xi(x)dx = a^2, \quad E\eta(t)^2 = \int_{-\infty}^{+\infty} x^2 f_\eta(x)dx = b^2,$$

$$\mathrm{Var}(\xi(t)) = E\xi(t)^2 - [E\xi(t)]^2 = a^2, \quad \mathrm{Var}(\eta(t)) = E\eta(t)^2 - [E\eta(t)]^2 = b^2,$$

$$\rho = \frac{E\xi(t)\eta(t) - E\xi(t)E\eta(t)}{\sqrt{\mathrm{Var}(\xi(t))\mathrm{Var}(\eta(t))}}, \quad |\rho| \leqslant 1 \tag{3}$$

where a and b are the intensity or the standard deviation of $\xi(t)$ and $\eta(t)$ respectively, and the parameter ρ ($|\rho| \leqslant 1$) measures the linear relationship strength between $\xi(t)$ and $\eta(t)$. Larger $|\rho|$ means stronger linear relationship between $\xi(t)$ and $\eta(t)$, while $\rho = 0.0$ means there are no linear relationship between $\xi(t)$ and $\eta(t)$.

3. Input periodic signal

In this section, to system (1)–(3), we will depict SR phenomenon by employing SNR theory when the input signal $s(t)$ is periodic.

3.1. The expression of the output SNR

Now, in Eqs. (1)–(3), we take $s(t)$ as the form of $s(t) = A\sin\omega t$, that means $s(t)$ is a sinusoidal signal with period $T_s = \frac{2\pi}{\omega}$, and ω is the frequency, A is the amplitude. Generally, in Eqs. (1)–(3), since the input information $s(t)(1+\xi(t)) + \eta(t)$ is the combination of the periodic force $s(t)$ and the random forces $\xi(t)$ and $\eta(t)$, so the output signal $y(t)$ is a cyclostationary random signal, and the transmission characteristic of $s(t)$ by system (1)–(3) is assessed by the output SNR which is a standard measurement in SR studies [11–14]. Moreover, the output SNR is defined as the power contained in the output spectral line at the fundamental $1/T_s$ divided by the power contained in the noise background in a small frequency band ΔB around $1/T_s$ [11–14], as following

$$\mathrm{SNR} = \frac{|\overline{Y_1}|^2}{\langle \mathrm{var}[y(t)]\rangle \Delta t \Delta B}, \tag{4}$$

where $|\overline{Y_1}|^2$ denotes the power of $y(t)$ contained in the output spectral line at the frequency $1/T_s$, |...| shows absolute value, and $\overline{Y_1}$ is the Fourier coefficient at the fundamental of the T_s-periodic nonstationary output expectation $E[y(t)]$, i.e.