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Concept, analysis, and demonstration of a novel delay network exhibiting stochastic resonance induced by external noise

Yukihiro Tadokoro [a,*], Seiya Kasai [b], Akihisa Ichiki [c]

[a] Toyota Central R&D Labs., Inc., Nagakute, Aichi 480-1192, Japan
[b] Research Center for Integrated Quantum Electronics, Hokkaido University, Sapporo, Hokkaido 060-8628, Japan
[c] Green Mobility Collaborative Research Center, Nagoya University, Nagoya, Aichi 464-8603, Japan

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ABSTRACT

Stochastic resonance offers the possibility of signal amplification by the addition of noise. This curious, interesting phenomenon has received considerable attention since the 1990s. Since such effect has the potential to improve the signal processing performance, intensive works have been done about this topic. One of the most effective implementations of stochastic resonance is in the Collins network, which can provide outstanding performance in that the network output consists of an amplified version of a weak, sub-threshold signal. In practical situations, the sub-threshold signal is easily buried in external noise from the environment. The present paper focuses on the discrete-time system (plus continuous-time system) and analyzes this situation to clarify the performance degradation of the amplification effect. As a countermeasure, we herein propose a novel delay network. The present analysis indicates that the proposed scheme produces an amplification effect in the presence of external noise. The results of the analysis are used to determine the condition for which the delay network is effective, and the results of an experimental evaluation verifies the validity of the analysis.

1. Introduction

Stochastic resonance (SR) is an interesting phenomenon that describes a constructive action of noise. More than two decades ago, SR was introduced in the context of nonlinear physics [1]. Stochastic resonance can enhance the system response in the presence of noise [2–4], and the effect of SR has been observed in various fields, such as biology [5–7], optics [8], and electronic circuits, including nano-devices [9–11].

In the signal processing field, one of the major topics regarding SR is the amplification of weak signals when noise is unavoidable. It is well known that a system with nonlinearity exhibits SR. In this sense, many works consider nonlinear devices/systems, and an attempt is made to detect the weak signal, which many authors refer to as a sub-threshold signal in threshold devices, by adding noise. Traditional nonlinear devices such as comparators and Schmitt triggers have been considered, and the effect of SR have been discussed [2,3,12,13]. Many papers on physics have measured the amplification gain, mainly by using the signal-to-noise ratio [1–3,6,14,15]. Since the goal in signal processing is the detection of a weak signal and/or the transmission of information, some studies have discussed the achieved effect in terms of signal detection performance, including miss-detection and false-detection probabilities [16–19], Fisher information [20] and mutual information [21,22]. The dependence of gain on the noise characteristics has been discussed in [23], and design methods to improve the detection have been reported [24–26]. It is worth mentioned here that signal processing algorithms exhibit the SR effect. S. Kay and his collaborators focused on the hypothesis testing problem, and performance gain by adding noise has been discussed [16,17,23,24]. Decision making is a nonlinear operation, so it exhibits SR. In the same period, F. Chapeau-Blondeau and his collaborators did extensive work focusing on the SR effect in optimum detection including Bayesian estimation and minimax detection [27–33]. Such investigations have made it clear that adding noise makes a contribution to an improvement in the performance of algorithms, which motivates one to apply the essence of SR to Neyman–Pearson detection [13], parameter estimation [22], nonparametric detection [34,35], and the forward error correction scheme [36]. These fundamental studies are good references for the application of SR in the fields of image processing [21,37] and communication systems [38].

To increase the SR gain, some studies have focused on establishing an effective SR device: tuning the device parameters including the threshold value [39] and full designing of the device

* Corresponding author. Fax: +81 561 63 6119.
E-mail address: tadokoro@mosk.tytlabs.co.jp (Y. Tadokoro).

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[5,9,12,24,40,41]. One method involves the use of an arrayed system [5,9,20,40,42]. The Collins network is a promising example [5], and theoretical analysis has revealed that with a sufficiently large number of nonlinear devices and identical and independently distributed (i.i.d.) noise sources, the system can output the sub-threshold signal itself. Another advantage is the "without tuning" effect. Generally, in order to take advantage of SR, the noise intensity should be tuned. However, this is not necessary in the case of the Collins network. Such advantages have been investigated in many fields, such as biology, neural science [5], nano-devices [9,10], and signal processing [29,32,38,40].

The present paper points out the problems with the Collins network in practical applications. To consider the digital signal processing, we focus on a discrete-time system in the analysis, whereas the essence will be discussed in a continuous-time system. The Collins network may encounter the following problems:

1. The performance deteriorates when the input to the network is corrupted by noise.
2. Independent noise should be added to the input signal for each device in the network. The performance can be enhanced by increasing the number of noise sources, but this leads to a complicated system.

In practical situations, the sub-threshold signal input to the network is easily buried in external noise such as thermal noise from surrounding equipment or switching noise in a power source. When the input is corrupted by such noise, the Collins network should amplify the noise as well as the sub-threshold signal, and as a result, the signal itself cannot be obtained at the output. Although the original study [5] did not consider this situation, some authors have analyzed the influence of white Gaussian noise [40]. The first contribution of the present study is an expansion of the analysis to arbitrary white noise, including non-Gaussian noise. The role of such non-Gaussian noise has recently become important [41,43,44]. Some studies have found that the application of SR is effective for non-Gaussian noise cases [32,41].

The second contribution of the present paper is the proposal of a novel delay network, which is effective in cases of external noise. The concept of the proposed method was briefly introduced in [45] and the performances were evaluated simply by numerical simulation. The present paper analytically describes the concept, and gives the detailed analysis of the proposed method. Such analysis contributes to make clear the characteristics of the proposed scheme, including the condition in which the proposed method is effective and the performance dependence on both the signal shape and the sampling frequency. The experimental results ensures the validity of the proposed method.

The essence of the delay network is the exploitation of a large number of delayed noise. The delayed version of the external noise is uncorrelated with the original noise when the delay is larger than the autocorrelation time for the noise. Such a delay realizes the i.i.d. noise which is included in the Collins network, so that the original sub-threshold signal can be outputted. In the present paper, the concept of the delay network is analytically introduced, and its performance is analyzed to verify its advantages. In the analysis, we focus on a static (memoryless) device, such as a comparator. An A/D converter, which is often used in digital signal processing, is one example of a comparator, and the quantization error will be reduced by utilizing the proposed method as well as the Collins network. The effectiveness of the delay network is also demonstrated experimentally. Note that such a network can have a simple structure. The uncorrelated noise is obtained from the external noise, which means that the delay network itself does not need to contain any noise sources.

The remainder of the present paper is organized as follows. In Section 2, a continuous-time system is considered, and the essence of the problem with the Collins network and the proposed delay network are analytically described. The performance of these networks is analyzed in detail and evaluated numerically in Section 3. For a simple analysis and the application to digital signal processing field, we consider a discrete-time system in this section. The sub-threshold signal is assumed to be square pulse and sampled at a high frequency compared to the Nyquist rate. The analysis clarifies the condition for which the delay network is effective in the white noise case. The situation of correlated noise may also occur, which is assumed in the demonstration in Section 4, and the performance is evaluated for this case. Finally, conclusions are presented in Section 5.

2. Problems with the Collins network and their solutions: Delay network

This section focuses on a continuous-time system and reviews the Collins network, points out the problems associated with this network, and then introduces the delay network as a countermeasure. The Collins network is a system in which an amplified version of a sub-threshold input signal can be obtained as the network output $y(t)$ [5]. The network consists of N parallel nonlinear devices $h(x)$, as shown in Fig. 1(a). In order to simplify the discussion, we consider a static (i.e., memoryless) device. Since the characteristics of the nonlinear devices are identical and they have a threshold, the sub-threshold signal $s(t)$ does not appear at the output. In order to obtain the SR effect, the input is added to an i.i.d. internal noise $\eta_i(t)$, where the subscript i indicates the index of the arrayed nonlinear device. In this addition, the resulting signal can exceed the threshold so that the device can output the sub-threshold signal component. The output of the ith nonlinear device can be expressed simply as

$$y_i(t) = h\big(s(t) + \eta_i(t)\big) = y^s(t) + y_i^\eta(t) \tag{1}$$

where $y^s(t)$ is the signal component and $y_i^\eta(t)$ is the noise component at output i. The signal and noise parts are given by $y^s(t) = \langle h'(\eta_i(t))\rangle s(t)$ and $y_i^\eta(t) = y_i(t) - y^s(t)$, respectively, where $h'(x)$ is the derivative of $h(x)$ and $\langle\cdot\rangle$ is the expectation. Obviously, Eq. (1) is valid under the condition $|y^s(t)| \approx 0$. Stochastic Taylor expansion [26] gives $y^s(t) = \langle h'(\eta)\rangle s(t)$, which implies that the condition is satisfied in two cases: $|s(t)| \approx 0$ and/or $\langle h'(\eta)\rangle \approx 0$. It is noteworthy that in the latter case, that is, when the characteristic of the nonlinear device has little change such as a comparator, Eq. (1) holds regardless of the input signal $s(t)$. Actually, because of the nonlinear effect, the output is usually written in a complicated form, rather than the simple form of the second equation of Eq. (1). To state our idea briefly, in the following, we consider the linear-response regime, the output of which can be a linear combination of the signal and the noise.

Due to the noise component $y_i^\eta(t)$, the output $y_i(t)$ should be noisy. The concept of the Collins network is smart because this network eliminates the noise simply by summing the output. Adding i.i.d. noise means that the noise component $y_i^\eta(t)$ is uncorrelated with the noise from other nonlinear devices, i.e., $\langle y_i^\eta(t) y_j^\eta(t)\rangle - \langle y_i^\eta(t)\rangle\langle y_j^\eta(t)\rangle = 0$ for $i \neq j$. In this situation, the averaging of the numerous uncorrelated noise components $\sum_{i=1}^N y_i^\eta(t)/N$ is equal to the expectation $\langle y_i^\eta(t)\rangle$. Note that $\langle y_i^\eta(t)\rangle$ is uncorrelated with the subscript i because of the uncorrelated property of $y_i^\eta(t)$. Then, the network output can be written as follows:

$$y(t) = \frac{1}{N}\sum_{i=1}^{N}\big\{y^s(t) + y_i^\eta(t)\big\} = y^s(t) + \big\langle y_i^\eta(t)\big\rangle. \tag{2}$$