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Detectors based on stochastic resonance, Part 2: Convergence analysis and perturbative corrections

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Abstract

This paper considers convergence properties and perturbative corrections for stochastic resonant (SR) detectors operating in a realistic marine environment. The description of such a detector is reviewed in the weak signal limit. A deterministic algorithm is developed to globally optimize the performance of the SR detector. It is established that this algorithm converges with logarithmic complexity. Scaling arguments demonstrate an improvement over standard, deterministic algorithms in two important limiting cases: (i) increasing accuracy of the optimization procedure and (ii) increasingly heavy-tailed marine noise probability density functions (PDFs), corresponding to increasingly turbulent ocean conditions. Perturbative corrections due to small nonzero input SNRs, temporal drift in the marine noise PDF and the effect of uncertainty in signal frequency are derived. The correction due to frequency error is found to be of second order, and hence subdominant to the former two which are of first order. For a class of marine noise PDFs, these corrections are expressed in terms of standard mathematical functions which are easy to compute in real-time. Numerical simulations indicate that the SR detector is stable under the perturbative corrections considered and that finite input SNRs constitute the dominant perturbative effect for standard ocean acoustic scenarios. It is stressed that these results imply efficient and inexpensive upgrades to existing sonar hardware.

Keywords: Stochastic resonance; Quantizer; Detector; Marine noise; Convergence rate; Perturbative analysis; Global optimization

1. Introduction

The phenomenon of stochastic resonance refers to the enhancement of signal transmission in certain nonlinear systems due to the addition of noise [1–5]. Originally, stochastic resonance was studied in threshold systems driven by weak, sub-threshold signals. New studies rooted in information theory [6–8] extend the range of this phenomenon to multi-threshold systems driven by supra-threshold signals. A study of this phenomenon in quantizers is given in [9,10]. Quantizers have since been shown to demonstrate an improvement over conventional matched filters for the detection of signals corrupted by non-Gaussian noise [9–17]. For reviews of quantization refer [18,19]. Such a quantizer-detector was designed for the detection of weak, sinusoidal signals in a marine environment [20]. Physically, this corresponds to a remote target to be detected. The detector consists of a stochastic resonant (SR)

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quantizer followed by a correlator. The SNR gain must be optimized in the appropriate asymptotic limit, ensuring that the detector works in the peak of its SR regime. Hence, we refer to it as the SR detector. A few practical issues that arise when operating such a SR detector in a marine environment are as follows:

(1) The value of the global maximum of the SNR gain G, in the limit as the input SNR $R_{\text{in}} \to 0$, and the corresponding quantizer threshold v_t must be determined for such a detector. Can the computational efficiency of the global optimization be quantified and, if possible, be improved in order to meet stringent hardware constraints?
(2) In practice, detectors operate when the input SNR R_{in} is small but nonzero. Can the resultant corrections to the SNR gain G and quantizer threshold be computed?
(3) The SNR gain G depends on the probability density function (PDF) governing the noise of the ocean. The PDFs of marine noise are marked by temporal variation. Can the resultant margin of error of G be explicitly computed?
(4) It is assumed that the signal to be detected, emitted by a target vessel, is a sinusoid of known frequency. The far-field acoustic radiation of most modern target vessels is dominated by the low frequency sinusoid emitted by the propellor. Higher frequency components due to cavitation and internal machinery are more severely attenuated by the marine environment. However, the uncertainty in the frequency of the dominant component is ~5%. Can the resultant degradation in detection performance due to an error in the estimate of the signal frequency be quantified?

This paper addresses the issues above and should be considered as an adjunct to [20], though we strive to be self-contained. The issue of the relative stability of 2-level and 3-level SR quantizers is addressed in [21]. The remainder of the paper is organized as follows. Section 2 contains a review of the necessary terminology, Sections 3–6 address (1–4), respectively, and Section 7 contains the conclusions and future extensions to this work.

2. Review: stochastic resonance in 3-level quantizers

Our treatment of quantizers exhibiting stochastic resonance is rooted in the SNR gain formalism given in [9]. The quantizer is driven by a sequence $u[n]$, which is assumed to be a sinusoid of unknown amplitude A_1, unknown phase ϕ and corrupted by additive noise as given by

$$u[n] = A_1 c_n + \sigma w[n], \tag{1}$$

where $c_n = \cos(2\pi n/N - \phi)$ and $w[n]$ are zero-mean, unit-variance independent and identically distributed (iid) random variables with PDF $f(\xi)$ and a cumulative distribution function $F(\xi)$. The output sequence of the quantizer is denoted by $y[n]$. The SNR gain of such a threshold system G, as defined in [9], is

$$\widetilde{G} = \frac{4|Y_1|^2}{A^2\overline{\sigma_y^2}}, \tag{2}$$

where Y_1 and $\overline{\sigma_y^2}$ denote the first Fourier coefficient of the sequence $E(y[n])$ and the average of the sequence $\sigma^2(y[n])$ observed over one period, respectively. A is the normalized amplitude given by $A = A_1/\sigma$. For a symmetric 3-level quantizer with thresholds $-v_t, v_t$ and quantization levels $-1, 0, 1$, the SNR gain then has the following Taylor series expansion to the first nonvanishing power of A:

$$\widetilde{G}(x) = G(x) + g_1(x)A^2 + \mathrm{O}(A^4), \tag{3}$$

where the dominant term is given by

$$G(x) = \frac{2f^2(x)}{1 - F(x)} \tag{4}$$

and the first nonvanishing correction, subdominant to $G(x)$, can be written as

$$g_1(x) = \frac{1}{2}\left(\frac{f(x)f''(x)}{1 - F(x)}\right) + \frac{1}{4}\left(\frac{f'(x) + 4f^2(x)}{1 - F(x)}\right)G(x). \tag{5}$$

In the above formulae x denotes the normalized threshold given by $x = v_t/\sigma$. The details are outlined in Appendix A.

3. The unperturbed functional $G(x)$

In the weak signal limit, when $R_{\text{in}} \ll 1$, the expansion of the Gain in (3) has an appealing and useful explanation in perturbation theory,

(1) $G(x)$, the unperturbed functional, dominates the 'landscape' of $\widetilde{G}(x)$, and therefore largely determines the maxima of the gain, and the values of the normalized threshold (x) where they are attained.