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Design and performance analysis of a signal detector based on suprathreshold stochastic resonance

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ABSTRACT

This paper presents the design and performance analysis of a detector based on suprathreshold stochastic resonance (SSR) for the detection of deterministic signals in heavy-tailed non-Gaussian noise. The detector consists of a matched filter preceded by an SSR system which acts as a preprocessor. The SSR system is composed of an array of 2-level quantizers with independent and identically distributed (i.i.d) noise added to the input of each quantizer. The standard deviation σ of quantizer noise is chosen to maximize the detection probability for a given false alarm probability. In the case of a weak signal, the optimum σ also minimizes the mean-square difference between the output of the quantizer array and the output of the nonlinear transformation of the locally optimum detector. The optimum σ depends only on the probability density functions (pdfs) of input noise and quantizer noise for weak signals, and also on the signal amplitude and the false alarm probability for non-weak signals. Improvement in detector performance stems primarily from quantization and to a lesser extent from the optimization of quantizer noise. For most input noise pdfs, the performance of the SSR detector is very close to that of the optimum detector.

1. Introduction

Optimal and robust detection of signals in non-Gaussian noise is a problem of great interest in several applications such as sonar [1,2], radar [3] and watermark detection [4]. Optimal detection of a deterministic signal $As(t)$ in additive white noise $i(t)$ involves computation of the following test statistic [5]

$$T_{OD}(x(t)) = \sum_{t=0}^{N-1} \ln([f_i(x(t) - As(t))]/f_i(x(t))), \tag{1}$$

where $\boldsymbol{x} = [x(0) \ \dots \ x(N-1)]^T$ is the data vector and $f_i(.)$ is the probability density function (pdf) of noise. The test statistic of the locally optimum (LO) detector for the detection of a weak signal ($A \to 0$) is generated by the nonlinear correlator

$$T_{LO}(x) = \sum_{t=0}^{N-1} L_i(x(t))s(t), \tag{2}$$

where

$$L_i(x) = -(df_i(x)/dx)/f_i(x) \tag{3}$$

is a memoryless transformation which may be called the locally optimal (LO) transform. (In literature, this

Abbreviations: SSR, Suprathreshold stochastic resonance; SD, Standard deviation; OD, Optimum detector; LOD, Locally optimum detector; NQ, Noisy quantizer; GM, Gaussian mixture; GG, Generalized Gaussian; CGM, Cauchy–Gaussian mixture; ST, Student's t-distribution; NP, Neyman-Pearson; SR, Stochastic resonance

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transformation is also referred to as the zero memory nonlinear transformation). Both these detectors reduce to a simple replica correlator or matched filter if the noise is Gaussian. But if the noise is non-Gaussian, the transformations defined in (1) and (2) are nonlinear functions and their implementation becomes a computationally difficult task. These detectors also require the prior knowledge of the noise pdf, and hence their performance is sensitive to errors in modeling the noise pdf. It is therefore of interest to design near-optimal detectors which are easy to implement and robust with respect to noise modeling errors.

A strategy that has been widely used for the design of simple suboptimal detectors in non-Gaussian noise is based on optimal quantization to approximate the nonlinear transformation associated with the optimal detector [6–9]. This approach is prompted by the fact that the LO transform $L_i(x)$ reduces to a simple one-bit quantization if the noise is Laplacian. Another approach that has received much attention in recent years involves a constructive use of noise to aid detection [10–24]. This approach is based on the phenomenon of stochastic resonance (SR) [25], which may be defined as a non-monotonic variation of a system performance measure with respect to input noise intensity. The performance measure that is of interest to us is probability of detection, although other performance measures such as output SNR [25,26], SNR gain [14,16], mutual (Shannon) information [27], Kullback entropy [28], Fisher information [29] and cross-correlation [30] have also been considered in other applications of SR. SR is exhibited by dynamic bistable systems and also by static (memoryless) systems with threshold nonlinearity [26]. A two-level quantizer is the simplest example of the latter class of systems. SR may be realized either by adding noise or by tuning the quantizer threshold [31]. But the realization of SR is limited to situations wherein the signal alone is too weak either to trigger a transition from one state to another or to advertise its presence in some other fashion. Stocks [32] showed that the weak signal constraint can be overcome if a single quantizer is replaced by an array of quantizers and the primary input is supplemented by an additional independent noise at each quantizer. The additional noises at the quantizer inputs are independent and identically distributed (i.i.d), and the output of the array is obtained by averaging the outputs from all quantizers. This extended version of SR is called suprathreshold stochastic resonance (SSR). Detectors based on a noisy quantizer array retain the inherent simplicity of quantizer-based detectors and achieve better performance by utilizing the diversity provided by the injected noises.

The use of SSR for detection of deterministic signals in non-Gaussian noise was first proposed by Rousseau et al. [18]. They proposed a simple test statistic obtained by correlating a replica of the signal with the output of the SSR system. Chen et al. [20] have derived sufficient conditions for improvability of performance of a fixed detector by adding noise. Chen and Varshney [21] have considered the SR noise optimization problem for detectors with variable quantizer threshold. Patel and Kosko [22] have derived necessary and sufficient conditions for the existence of NP-optimal SR noise. In a subsequent paper [23], they have also determined necessary and sufficient conditions for noise-enhanced detection of deterministic signals in non-Gaussian noise using quantizer arrays. Guerriero et al. [24] have studied SR effect in the context of sequential detection for shift-in-mean problems. The benefits of SR also extend to noisy quantizer array-based linear mean-squared error estimation [33].

In this paper we present details of the design and performance analysis of the nonlinear correlator-detector based on SSR [18]. A preliminary version of this paper with a limited treatment of the topic appeared in [19]. The motivation for this work stems from the need to avoid the computational complexity associated with the optimal and LO detectors. The SSR detector can be easily implemented using an array of noisy one-bit quantizers and a correlator. Design of the SSR detector involves optimization of the variance and the pdf of quantizer noise. The present paper is organized as follows. The mathematical model of the SSR detector is presented in Section 2. An expression for the NP-optimal variance of quantizer noise is derived in Section 3 for the case of weak signal detection. It is shown that the optimal variance depends on the pdfs of input noise and quantizer noise. It is shown in Section 4 that the quantizer noise variance that maximizes the probability of detection also minimizes the mean-square difference between the output of the quantizer array and the value of the LO transform defined in (3). Simulation results are presented in Section 5 for four different models of input noise pdf. The issue of optimization of quantizer noise pdf is discussed in Section 6. Optimization of quantizer noise variance for detection of non-weak signals is discussed in Section 7. Conclusions are presented in Section 8.

2. SSR detector

Consider the detection of a signal $As(t)$ with a known waveform $s(t)$ and an unknown amplitude $A>0$, embedded in additive non-Gaussian noise $i(t)$. The data consists of N samples of $x(t)$ and the detection problem is cast as the following hypothesis testing problem

$$H_0 : x(t) = i(t)$$
$$H_1 : x(t) = As(t) + i(t), \quad t = 0,1,\ldots,N-1. \tag{4}$$

It is assumed that the noise samples $i(0), \ldots, i(N-1)$ are i.i.d random variables with zero mean and unit variance with probability density function $f_i(\xi)$ and cumulative distribution function (cdf) $F_i(\xi)$, and that $f_i(\xi)$ is an even function. It is also assumed that

$$\frac{1}{N}\sum_{t=0}^{N-1} s^2(t) = 1, \tag{5}$$

so that A^2 denotes the signal power.

The SSR detector consists of an SSR preprocessor, shown in Fig. 1, followed by a replica correlator or matched filter [18,19]. The SSR preprocessor is a parallel array of M one-bit quantizers. The input to the mth quantizer is $x(t)+\sigma q_m(t)$, where $\{q_m(t);\ m=1,\ 2,\ \ldots,\ M\}$