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Mechanical Systems and Signal Processing 21 (2007) 138–150
Mechanical Systems and Signal Processing
www.elsevier.com/locate/jnlabr/ymssp

Engineering signal processing based on bistable stochastic resonance

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Received 26 April 2005; received in revised form 28 June 2005; accepted 5 August 2005
Available online 22 September 2005

Abstract

The stochastic resonance (SR) characteristics of a single bistable system and two bistable systems connected in series with small and large parameters have been investigated, respectively. The viewpoint is that a single bistable system is better than a cascaded bistable system in detecting a weak periodic signal in frequency domain, and that, in time field, a cascaded system can detect a more beautiful waveform of either a periodic or an aperiodic weak signal. However, for some detection of a special signal, the periodic pulse for instance, a single bistable system is of great benefit to the signal extraction in time domain. It can provide some important information properties that are hardly obtained in frequency spectrum. Two examples of detecting a weak signal embedded in strong noise are presented in the end to illustrate that a single bistable system and a cascaded bistable system are both powerful tools for signal processing.

Keywords: Stochastic resonance; Bistable system; Noise; Power spectrum; Time waveform

1. Introduction

Stochastic resonance (SR) [1–6] phenomenon has been extensively paid attention to in fields such as weak signal detection [4], neural information coding [5], etc. It relates to a wide variety of physical systems [6,7], including monostable systems, multistable systems, and threshold systems. SR is commonly described as an increase in the signal-to-noise ratio (SNR) at the output of a non-linear dynamic system. This is obtained by varying the input noise level, but keeping the input modulation signal. In the background of a frequency field, SR means that, when a sinusoidal driving force mixed with noise is inputted into a non-linear system, a maximum spectral spike at the driving frequency of the system response spectrum can be viewed through varying the noise intensity. However, in some cases, especially for the detection of weak pulse or aperiodic signal, signal processing in time domain is necessary. To some extent, time waveform can reflect the dynamic behaviour of the tested object directly.

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0888-3270/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.ymssp.2005.08.002

At present, there has been a large amount of literature of SR study with a single bistable system on the amplification and recognition of a weak periodic signal [4,8–11]. Especially in the extraction of a weak sinusoidal signal submerged in strong noise, by the mechanism of the scale transformation stochastic resonance (STSR) [12,13] proposed by our group, we extend the traditional small parameter SR into large parameter SR. However, only a few documents [14] of SR discussion with two or more bistable systems connected in series exist in the research field. In the present work, the SR properties of a single bistable system and bistable systems connected in series will be investigated in detail. To further illustrate the function of bistable SR in signal processing in time field, under the condition of large parameters, an investigation is led to a comparable practical application, i.e., weak signal detection using both a single bistable system and a cascaded bistable system, in order to lay a foundation for the engineering application of the technique.

2. The SR of a single bistable system

The three basic ingredients of producing SR phenomenon are a bistable or multistable system, a weak coherent input (such as a periodic signal) and a source of noise that is inherent in the system, or that adds to the coherent input. For a convenient description, consider the overdamped motion of a Brownian particle in a bistable potential in the presence of noise and periodic forcing

$$\mathrm{d}x/\mathrm{d}t = -U'(x) + a\,\sin(2\pi f_0 t + \varphi) + n(t), \tag{1}$$

where $U(x)$ denotes the reflection-symmetric quartic potential

$$U(x) = -\tfrac{1}{2}\mu x^2 + \tfrac{1}{4}x^4. \tag{2}$$

Then Eq. (1) can be written as

$$\mathrm{d}x/\mathrm{d}t = \mu x - x^3 + a\,\sin(2\pi f_0 t + \varphi) + n(t), \tag{3}$$

where $n(t) = \sqrt{2D}\xi(t)$ with $\langle n(t)n(t+\tau)\rangle = 2D\delta(t)$ and noise intensity D, $\xi(t)$ presents a zero-mean, unit variance Gaussian white noise, μ is a real parameter, a is the periodic signal amplitude and f_0 is the modulation frequency. Eq. (3), a bistable system subject to sinusoidal signal and white noise, is the non-linear Langevin equation for one variable. For convenience, let $\varphi = 0$. Asymptotically $t_0 \to -\infty$, the memory of the initial conditions gets lost and the mean value $\langle x(t)|x_0, t_0\rangle$ becomes a periodic function of time. For small amplitudes, the response of the system to the periodic input signal can be written as

$$\langle x(t)\rangle_{as} = \bar{x}\sin(2\pi f_0 t - \bar{\varphi}) \tag{4}$$

with amplitude $\bar{x}$ and a phase lag $\bar{\varphi}$ which have approximate expressions, respectively, as

$$\bar{x} = \frac{\mu E[x^2]}{D}\frac{r_k}{\sqrt{r_k^2 + \pi^2 f_0^2}}, \tag{5a}$$

$$\bar{\varphi} = \arctan(\pi f_0 / r_k), \tag{5b}$$

where r_k is Kramers rate [2]

$$r_k = \frac{1}{\sqrt{2}\pi}\exp\left(-\frac{\Delta U}{D}\right) \tag{5c}$$

and $\langle x^2\rangle_0$ is the D-dependent variance of the stationary unperturbed system ($A_0 = 0$)). ΔU is the height of potential barrier between wells. In terms of [2,15], the most important feature of the amplitude $\bar{x}$ is that it depends on the noise strength D, i.e., from Eq. (5) the amplitude $\bar{x}$ first increases with increasing noise level, reaches a maximum, and then decreases again. This is the celebrated SR effect shown in Fig. 1. At a closer inspection of Fig. 1, it is also important to note that the variation of the frequency f_0 at fixed D does not yield a resonance-like behaviour of the response amplitude. The behaviour of $\bar{x}$ versus increasing f_0 at fixed D is generally that of a monotonically decreasing function. The response amplitude $\bar{x}$ becomes very small with the increase of f_0. This point illustrates that SR phenomenon requires a low driving frequency, i.e., small parameter frequency.