Commun Nonlinear Sci Numer Simulat 18 (2013) 2852–2858

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Suprathreshold stochastic resonance in multilevel threshold system driven by multiplicative and additive noises

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ARTICLE INFO

Article history:
Received 12 January 2012
Received in revised form 19 December 2012
Accepted 7 February 2013
Available online 18 February 2013

Keywords:
Suprathreshold stochastic resonance
Multilevel threshold system
Mutual information
Multiplicative and additive noises

ABSTRACT

The suprathreshold stochastic resonance in multithreshold neuronal networks system driven by multiplicative Gaussian noise and additive Gaussian noise is studied. The expression of the mutual information is derived, and the effects of the noise intensity and system parameter on mutual information are discussed. It is found that adjusting the additive noise intensity is more effective than adjusting the multiplicative noise intensity to enhance information transmission, and the more the number of devices, the more apparent the phenomenon of suprathreshold stochastic resonance. Moreover, we also found that the selection of threshold is very important in the process of information transmission.

1. Introduction

Stochastic resonance (SR) is a nonlinear phenomenon which describes the action of noise improve the performance of a signal-processing system [1]. In recent years, study of SR in threshold based system has received considerable attention. In such systems, the addition of noise can lead to an enhancement of the system's response to weak subthreshold signals [2,3]. In 2000, an interesting new form of SR has been introduced under the name of suprathreshold stochastic resonance (SSR) [4]. SSR has been observed in summing arrays of threshold elements, it is characterized by a noise-induced maximum in the transmitted signal information and not restricted to a small or subthreshold signal.

In Refs. [5–18], SSR was shown to occur in the Shannon average mutual information between the input and output of the array. Like all forms of stochastic resonance, this means that the output performance is maximised by nonzero input noise. Since SSR was first demonstrated in Ref. [4], SSR has been applied to arrays of sensory neurons, to motion detectors, to cochlear implants, or to signal estimation tasks [7]. Although SSR has now been studied in a wide variety of different contexts, all these studies have been undertaken assuming that the noise is additive or multiplicative. However, it is well established that in neural systems, the noise may enter in both additive and multiplicative ways [7,19]. Thus, the neural systems with additive and multiplicative noises need to be studied.

In this paper, the SSR in multithreshold neuronal networks system driven by multiplicative Gaussian noise and additive Gaussian noise is investigated. The expression of the mutual information is derived, and the effects of the noise intensity and system parameter on mutual information are discussed.

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http://dx.doi.org/10.1016/j.cnsns.2013.02.006

2. Multithreshold neuronal networks model with multiplicative noise and additive noise

Although there are a number of studies of SR that consider SR with additive and multiplicative noises [19–21], few have considered an array of such devices (a notable exception is the study by Ref. [7]) and, as yet, there have been no studies of SSR with additive and multiplicative noises. According to Refs. [7,19], the N-threshold neural network model with additive and multiplicative noises is: (see Fig. 1).

In this system, the output signal y of each unit can be expressed as a Heaviside function [7]

$$y_i = \begin{cases} 1, & v_i \geqslant U_i \\ 0, & v_i < U_i \end{cases}, \tag{1}$$

The input to each threshold unit is given by

$$v_i = \lambda F(x) + D\xi_i F(x) + Q\eta_i. \tag{2}$$

$i = 1,2,\ldots,N$, ξ_i, η_i are independent Gaussian noise, and meet to

$$\langle \xi_i \xi_j \rangle = 0, \quad \langle \eta_i \eta_j \rangle = 0, \quad \langle \xi_i \eta_j \rangle = 0, \quad i \neq j \quad i,j = 1,2,\ldots,N \tag{3}$$

and D, Q are multiplicative and additive noise intensity, λ is the deterministic signal strength of the coupling in the system, U_i is the threshold of unit for the system, x is input signal of the system. Here we selected x as Gaussian, and its probability density is

$$P_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right). \tag{4}$$

When $D \to 0$ the model transformed into the case of Refs. [3–5], there only with additive noise, and when $Q \to 0$ the model transformed into the case of Ref. [7], there only with multiplicative noise.

Each threshold unit was subject to the same input signal x, the total output of the system may obey the binomial distribution. The probability density function of output is given as follows [7]:

$$\mathrm{P}\left\{\sum_{i=1}^{N} y = m|x\right\} = C_N^m q_x^m (1-q_x)^{N-m}, \tag{5}$$

here

$$C_N^m = \frac{N!}{m!(N-m)!}, \qquad q_x = P\{y_i = 1|x\}. \tag{6}$$

For the purposes of simplifying the calculation, we assume that the probability density of Gaussian noise ξ_i, η_i are subject to the standard Gaussian distribution, and all units had the same thresholds, that is $U_1 = U_2 = \cdots = U_N = U$ [7].

In the following discussion, we take the linear function $F(x) = x$, therefore,

$$v_i = \lambda x + D\xi_i x + Q\eta_i(t). \tag{7}$$

In the case of input signal x and additive noise and multiplicative noise present, the conditional probability density of v_i was Gaussian, and its expression is

$$P_{v|x}(v_i|x) = \frac{1}{\sqrt{2\pi(D^2x^2+Q^2)}} \exp\left(-\frac{(v_i-\lambda x)^2}{2(D^2x^2+Q^2)}\right) \tag{8}$$

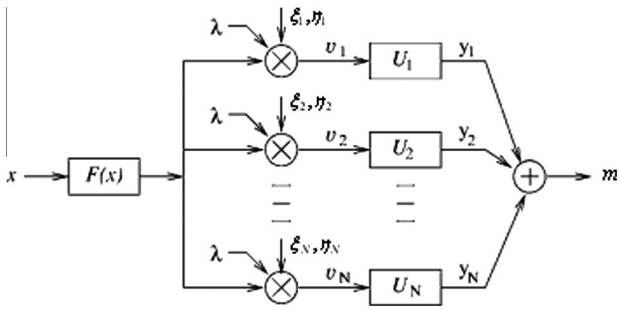

Fig. 1. N-threshold neural network model with additive and multiplicative noise [7].