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Study of frequency-shifted and re-scaling stochastic resonance and its application to fault diagnosis

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ABSTRACT

When detecting a weak and high-frequency signal submerged in strong noise, the existing large parameter stochastic resonance (LPSR) models need either a high sampling frequency or a large number of sample points. To breach the above limits and raise the usability of LPSR, a novel method named frequency-shifted and re-scaling stochastic resonance (FRSR) is proposed in this paper. By shifting and re-scaling the frequency, FRSR provides a way to alleviate the contradiction between sampling frequency and the number of sample points. The proposed method is verified with simulated signals. The results show that this method is useful in weak fault diagnosis of mechanical systems which involve high feature frequencies. Compared with the existing LPSR models, FRSR just requires a lower sampling frequency, a smaller data length and has higher efficiency. Finally, the proposed method is applied to a milling machine tool fault diagnosis and an outer ring fault feature of the spindle bearing is found successfully. Thus, FRSR is a meaningful try for SR coming into practical use of mechanical fault diagnosis.

1. Introduction

The increasing demand of quality in production processing has encouraged the development of several studies of fault detection and diagnosis in industrial plants. However, users of modern measurement devices are annoyed by any source of background noise. Almost all conventional methods in signal processing are to filter or mask noise [1–4]. The usable signal may be weakened or even destroyed at the same time. In fact, noise itself is a signal and a free source of energy. And noise can amplify a weak signal in some nonlinear systems even though too much noise can swamp the signal [5,6]. The method of using noise to enhance signals is called stochastic resonance (SR) [5–9].

During the past two decades, there have been many theoretical developments of SR in bistable systems [7–11]. However, most of them focus on low frequency and weak periodic signals immersed in small noise, i.e., small parameter signals (the values of the frequency and amplitude of a periodic signal and noise intensity are all smaller than 1). It is because they were restricted by adiabatic approximation or linear response theory, both of which call for small parameters. But large parameter problems (the values of frequency and/or amplitude and/or noise intensity can be much larger than 1) may usually be involved in fault diagnosis of mechanical systems. Therefore, the study of large parameter stochastic resonance (LPSR) becomes necessary, and in fact several improvements have been achieved during the past few years, such as scale

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normalized stochastic resonance (SNSR) [12], modulated stochastic resonance (MSR) [13], re-scaling frequency stochastic resonance (RFSR) [14–16] and so on. Each of them can be used to deal with some kinds of large parameter problems by themselves. However, both SNSR and RFSR need a super-high sampling frequency f_s (at least 50 times of signal frequency f_0) [14]; MSR requires a small frequency scanning step and a large number of sample points. By shifting the target frequency to the base frequency band, Frequency-shifted and re-scaling stochastic resonance (FRSR) proposed in this paper allows the sampling frequency f_s to be much lower; by compressing the frequency scale linearly, FRSR allows the data length to be smaller, and frequency scanning step can be much bigger than MSR, which makes the method more efficient.

In the present work, the classical SR theory is introduced in brief in Section 2 and the novel method FRSR is proposed in Section 3. Then, simulation comparisons are carried out among the proposed method and the existing methods of LPSR (MSR and RFSR) in Section 4. Finally, FRSR is applied to fault diagnosis of a milling machine tool in Section 5. The comparison and application results prove the low cost and high efficiency of the proposed FRSR technique in fault diagnosis.

2. The classical SR

To produce an SR phenomenon, at least three ingredients are required: (1) a bistable or multistable system, (2) a weak coherent input (a periodic/aperiodic signal) and (3) a source of noise that is inherent in the system or that adds to the coherent input. Here, we consider the overdamped motion of a Brownian particle in a bistable potential in the presence of noise and periodic forcing

$$\dot{x}(t) = -V'(x) + A_0 \cos(\Omega t + \varphi) + \xi(t), \tag{1}$$

where $V(x)$ denotes the reflection-symmetric quartic potential

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4, \tag{2}$$

then Eq. (1) can be written as

$$\dot{x}(t) = ax - bx^3 + A_0 \cos(\Omega t + \varphi) + \xi(t), \tag{3}$$

In Eq. (3), the barrier parameters a and b are positive real parameters, A_0 is the periodic signal amplitude, Ω ($= 2\pi f_0$) is the driving frequency, $\xi(t)$ denotes a zero-mean, Gaussian white noise, i.e.,

$$\langle \xi(t) \rangle = 0, \tag{4}$$

$$\langle \xi(t)\xi(t+\tau) \rangle = 2D\delta(\tau). \tag{5}$$

Here, D is the noise intensity and $\langle * \rangle$ stands for the statistical mean value calculation.

For convenience, we choose the phase of the periodic driving $\varphi = 0$. Asymptotically ($t_0 \to -\infty$), the memory of the initial conditions gets lost and $\langle x(t)|x_0, t_0 \rangle$ becomes a periodic function of time, i.e., $\langle x(t) \rangle_{as} = \langle x(t+T_\Omega) \rangle_{as}$ with $T_\Omega = 2\pi/\Omega$. For small amplitudes, the response of the system to the periodic input signal can be written as

$$\langle x(t) \rangle_{as} = \bar{x} \cos(\Omega t - \bar{\phi}) \tag{6}$$

with amplitude $\bar{x}$ and a phase lag $\bar{\phi}$, approximate expressions for which read

$$\bar{x}(D) = \frac{A_0 \langle x^2 \rangle_0}{D} \frac{2r_K}{\sqrt{4r_K^2 + \Omega^2}}, \tag{7a}$$

$$\bar{\phi}(D) = \arctan\left(\frac{\Omega}{2r_K}\right). \tag{7b}$$

r_K is the Kramers rate: $r_K = 1/\sqrt{2}\pi \exp(-\Delta V/D)$. An important feature of the amplitude $\bar{x}$ is that it depends on the noise strength D. In fact, according to Eq. (7a), by enhancing noise level, the amplitude $\bar{x}$ will first increase and then decrease after reaching a maximum. This celebrated SR effect is shown in Fig. 1. From Fig. 1, it is easy to find that the variation of the frequency f_0 for fixed noise intensity D does not always yield a resonance-like behavior of the response amplitude. For fixed noise intensity D, as f_0 increases the response amplitude $\bar{x}$ decreases sharply. This indicates that SR phenomenon requires a low-driving frequency or that SR enhances low-frequency signal much more than high frequency ones. For more details of why SR can only deal with low-driving frequency problem, please refer to works of adiabatic approximation theory [17] and linear response theory [18].

To show the advantage of SR in detecting the weak signal buried in heavy noise (the noise is ‘heavy’ enough to swallow the weak signal, but the intensity of the noise is smaller than 1), a simulation is done, shown in Fig. 2, in which the relative parameters corresponding to Eq. (3) are $A_0 = 0.1$, $f_0 = 0.02$, $D = 0.7$, $\varphi = 0$, $a = 0.4$, $b = 0.2$. The sampling frequency is $f_s = 3$. The data length is 1500 points; and all of the SR output points are used to calculate the frequency spectrum. For a better view in frequency domain, only low-frequency band is displayed, in which f_0 is included. Eq. (3) is numerically solved with a fourth-order Runge–Kutta discretization (in this paper, all the following numerical calculations are processed