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Correlated probabilistic load flow using a point estimate method with Nataf transformation ☆

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ABSTRACT

Wind speed as well as the power output of wind turbine generators (WTGs) have high correlations and may not be normally distributed. In this paper, the method of Zhao's point estimate method (PEM) combined with Nataf transformation was applied into correlated probabilistic load flow (PLF) calculation. This method can deal with correlated input random variables (RVs) with normal or non-normal probability distributions. Instead of joint probability density functions (PDFs) of multivariate RVs, this method only requires data of the marginal distribution function of each input RV and their correlation coefficients. The effectiveness of the proposed method is demonstrated by the numerical tests on IEEE 14-bus and the IEEE 118-bus systems. Besides, relative average errors compared with correlated Monte Carlo Simulation (CMCS) are analyzed.

Introduction

Wind power is one of the most important renewable energy resources. Wind turbine generators (WTGs) have seen considerable and growing worldwide developments during the past few decades. Increased wind power penetration brings about considerable challenges for power system operation and planning. The output of WTGs may fluctuate severely, and the forecasting errors are significant. Furthermore, the geographical location of wind farms (WFs), as well as their dependence on the wind speed, must be taken into consideration for power system operation schedule and network planning.

Probabilistic load flow (PLF) is an effective tool for power system analysis since it allows uncertain variables to be taken into account. The purpose of PLF is to obtain the probability density functions (PDFs) of the output variables considering the uncertainty of the input variables. The concept of PLF was first established by Borkowska [1]. These methods can be classified into three categories: simulation methods, analytical methods, and approximation methods. Simulation methods refer to Monte Carlo Simulation (MCS) models [2–4], where a large number of input variable samples are stochastically generated and, for each set of samples, a deterministic load flow calculation is performed. MCS is typically time-consuming, although it is possible to achieve results with a high degree of accuracy. A number of sampling techniques have been established to decrease the computational expense of MCS methods, including Latin hypercube sampling (LHS) [5,6], Latin supercube sampling [7], and importance sampling [8,9]. However, issues with computational expenses remain, and MCS methods are still used as the benchmark for other methods. Analytical methods are typically far less computationally expensive. Most analytical approaches to PLF are based on linearized [10] or multi-linearized [11] load-flow equations. The most commonly used analytical methods are the cumulant method [12] and the convolution method [13]. The cumulant combined with Gram–Charlier expansions has been applied to PLF; the resulting cumulative distribution curves of output random variables (RVs) are given in [12]. Discrete frequency domain convolution can also be used to obtain PLF solutions [13]. Alternatively, approximate methods are effective in reducing the computational expense of PLF problems compared with MCS. The main techniques in this category are the point estimate method (PEM) [14,15] and first-order second-moment method [16]. These methods approximate the statistical properties of the output variables. Other methods include the maximum entropy method with Gram–Charlier expansion, proposed for PLF applications in 2013 [17].

The methods discussed above do not consider the correlations of the input RVs. However, when correlations are taken into account, the solution to the PLF problem becomes more complicated. In the correlated MCS (CMCS) method, the most important

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Nomenclature

a	index for month
b	index for day in one month
h	index for wind speed sample on one day
H	number of samples on one day
r^{ij}_{a-b}	wind speed correlation coefficient on day b of month a between wind farm i and j
v^{i}_{a-b-h}	wind speed measurement of sample h on day b of month a in wind farm i
v^{j}_{a-b-h}	wind speed measurement of sample h on day b of month a in wind farm j
$\bar{v}^{i}_{a-b}$	wind speed mean value on day b of month a in wind farm i
$\bar{v}^{j}_{a-b}$	wind speed mean value on day b of month a in wind farm j
v_{in}	cut-in speed
v_{out}	cut-out speed
v_{rate}	rating speed
P_{rate}	rating wind power
P_w	WTG's output
L_1	independent standard normal space
L_2	correlated standard normal space
L_3	desired sample space
n	number of random variables
$\mathbf{X}$	output random vector in PLF
$\mathbf{Y}$	input random vector in PLF
$\mathbf{Z}$	random vector in space L_1
$\mathbf{W}$	random vector in space L_2
$\mathbf{C_Y}$	correlation coefficient matrix of $\mathbf{Y}$
$\mathbf{C_W}$	correlation coefficient matrix of $\mathbf{W}$
$\mathbf{S_Y}$	sample matrix of $\mathbf{Y}$
$\mathbf{S_Z}$	sample matrix of $\mathbf{Z}$
$\mathbf{S_W}$	sample matrix of $\mathbf{W}$
Φ	cumulate density function (CDF) of standard normal variable
F_i	CDF of input variable y_i
N^{-1}	inverse Nataf transformation
m	index for estimating point of each random variable
p_m	weighting coefficient for estimating point m
$\mu^{V_i}_{CMCS}$	voltage mean value for bus i using CMCS
$\mu^{V_i}_{PEM}$	voltage mean value for bus i using PEM
$\sigma^{V_i}_{CMCS}$	voltage standard deviation for bus i using CMCS
$\sigma^{V_i}_{PEM}$	voltage standard deviation for bus i using PEM
$\mu^{P_i}_{CMCS}$	branch power mean value for line i using CMCS
$\mu^{P_i}_{PEM}$	branch power mean value for line i using PEM
$\sigma^{P_i}_{CMCS}$	branch power standard deviation for line i using CMCS
$\sigma^{P_i}_{PEM}$	branch power standard deviation for line i using PEM
$\varepsilon^{V_i}_{\mu}$	relative error of voltage mean value for bus i
$\varepsilon^{V_i}_{\sigma}$	relative error of voltage standard deviation for bus i
$\varepsilon^{P_i}_{\mu}$	relative error of branch power mean value for line i
$\varepsilon^{P_i}_{\sigma}$	relative error of branch power standard deviation for line i
$\bar{\varepsilon}^{V}_{\mu}$	average relative error of buses voltages' mean value
$\bar{\varepsilon}^{V}_{\sigma}$	average relative error of buses voltages' standard deviation
$\bar{\varepsilon}^{P}_{\mu}$	average relative error of branches power' mean value
$\bar{\varepsilon}^{P}_{\sigma}$	average relative error of branches power' standard deviation

issue is to obtain a large number of correlated samples for each input RV. If the joint PDFs of multivariate RVs are known, then the corresponding samples can be achieved by random sampling. However, since the joint PDFs of multivariate RVs are difficult to obtain in practice, some probability information must be ignored. Most sampling techniques depend on the marginal PDF of each RV, as well as their correlation coefficients. Orthogonal transformation can be used to transform RVs from correlated normal space to an independent normal one. However, for multivariate correlated non-normal RVs, orthogonal transformation will result in significant errors. Liu used Cholesky decomposition and Nataf transformation to achieve mapping from correlated non-normal space to an independent standard normal space [18]. The sampling techniques combining Cholesky decomposition with Nataf transformation to handle correlated non-normal RVs are generally considered to be favorable. LHS can be used to generate samples of correlated non-normal RVs, and hence reduce the computational expense [19]. In the work reported here, a CMCS-based PLF model, in which Liu's technique [18] is adopted, is used as a benchmark tool for other approaches.

Neither the cumulant method nor the convolution method is inherently suitable for handling correlated RVs. Joint cumulants can be derived from joint PDFs of input RVs [20], and an orthogonal transformation has been used in the cumulants calculation to account for correlated input RVs [21].

PEM is widely used in power system PLF evaluation for the following reasons: linearization of the load flow equations is not required and it is computationally efficient. The most commonly used PEM is based on Hong's work [22]. However, this approach cannot handle correlated input RVs inherently. Therefore, some modifications to the original method are required. Harr combined Hong's PEM with orthogonal transformations to deal with correlations [23], and Morales applied Harr's idea to PLF study [24]. The unscented transform (UT) [25] was applied to power system PLF calculations. However, probability information higher than second order cannot be achieved. Other approaches, including Gaussian mixture models [26] and kernel estimators [27], have also been used to model correlated non-normal distribution functions in PLF study.

The main contribution of the work reported here is to apply the Zhao's PEM [28] accompanying Nataf transformation to power system correlated PLF evaluation. In contrast to the Hong's commonly used PEM formulism, Zhao's PEM can inherently handle the correlations in RVs [29,30]. Zhao's original PEM has been applied to power system PLF evaluations in [31]. However, the joint PDFs of the input RVs must be known in advance, which is not always practical. Here, the Rosenblatt transformation used in Zhao's PEM is replaced by the Nataf transformation, which only requires data for the marginal distribution function of each input RV and the correlation coefficients. The Nataf transformation is more practical than the Rosenblatt transformation and more accurate than orthogonal transformation when dealing with correlated non-normal RVs.

The remainder of this paper is organized as follows. Section 'Uncertainty modeling' describes the uncertainty models of load and wind power output. Section 'Zhao's PEM based on Nataf transformation' illustrates the method of Zhao's point estimate method and Nataf transformation. Two case studies are introduced in Section 'Discussion'. The modified IEEE 14-bus test system is used to demonstrate the accuracy of the proposed method while