Electrical Power and Energy Systems 31 (2009) 474–481

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Probabilistic load flow with wind production uncertainty using cumulants and Cornish–Fisher expansion

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ARTICLE INFO

Article history:
Received 26 September 2008
Accepted 17 February 2009

Keywords:
Probabilistic load flow
Wind generation
Cumulants
Cornish–Fisher expansion series

ABSTRACT

This paper proposes a method for probabilistic load flow in networks with wind generation, where the uncertainty of the production is non-Gaussian. The method is based on the properties of the cumulants of the probability density functions (PDF) and the Cornish–Fisher expansion, which is more suitable for non-Gaussian PDF than other approaches, such as Gram–Charlier series. The paper includes examples and comparisons between different methods proposed in literature.

1. Introduction

The great proliferation of wind energy in power networks has increased the uncertainty of power system operation and management. This uncertainty affects both the long and medium term system planning, and the day-ahead operation. In both cases, to have an assessment of the values of the system variables under these uncertain conditions is important for the Transmission System Operator. This is why the importance of probabilistic tools for power system analysis is increasingly growing. For long term studies, to consider the uncertainty in power planning may lead to a less expensive network. For the daily operation, an adequate assessment of the system variables may lead to a better management of congestions and other important advantages.

Among these tools, probabilistic power flow is one of the best known. To study all possible combinations of generation and load is impractical due to the great size of real networks, and this is why analytical methods should be used to adequately assess the variability of the grid magnitudes. From the first proposals in the seventies [1,2], a great deal of literature can be found about this subject. In these references, it was mainly the uncertainty of the load what was considered.

The most straightforward method of solving this problem is Monte Carlo simulation. This technique involves repeated simulation with values obtained from the PDF of the random variable considered. But for an adequate representation, a great deal of simulations must be considered in real systems. This makes this approach unpractical. One of the alternatives to this is the convolution of the PDF of the random variables involved, when they are independent of each other, and linearly related. Although this reduces the computational burden, it is still a formidable task to obtain the PDF of a single line when several random power injections are considered. Fast Fourier Transform (FFT) techniques were proposed to reduce the computational burden [3], but this method is linked to the convolution technique, and does not solve the problem efficiently. Recently, the use of cumulants and the approximation of resultant PDF by orthogonal series (Gram–Charlier expansion series) have been proposed [4]. It has interesting properties, and is computationally inexpensive.

Probabilistic load flow has been applied mostly to consider the uncertainty of load. The uncertainty of load is not very high, and it is frequently modeled using Gaussian probabilistic density functions. Wind energy proliferation, however, rends this approach insufficient, since the variability of wind power production is much higher, and usually the PDF are not Gaussian. Long term planning studies must consider PDF based on Weibull distributions, while short term operation analysis need to use PDF whose properties are still under study. Ref. [5] proposes the use of FFT and convolution in distribution networks, and makes estimates, under simplified conditions, of the PDF for short term wind power prediction. For large transmission networks it seems that the approach based on cumulants is very appropriate due to the low computational requirements. However, for non-Gaussian PDF, Gram–Charlier expansion series have serious convergence problems, and alternative tools have to be proposed. In this paper, an approach using Cornish–Fisher expansion is presented. This

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doi:10.1016/j.ijepes.2009.02.003

method has better convergence properties, without more computational burden.

The method is applied to next day operation of systems with great wind penetration. Usually, the system operators of these networks use the output of a short term wind power prediction program to forecast the next day grid constraints and congestions. However, these predictions have a lower accuracy than load forecasts, and their uncertainty should be also taken into account, for instance for the available interconnection capacity between systems. Therefore, a tool able to assess the probability of surpassing the allowed line capacity is very practical for an adequate operation.

The paper is structured as follows: first, a short view of short term wind power prediction, and the uncertainty associated to its output, is given. Then, the theoretical foundations of the Cornish–Fisher expansion series and the necessary definitions are introduced. The method is applied to a test grid, and its results are compared to those obtained using Gram–Charlier expansion series. A summary with the main conclusions ends the paper.

2. Short term wind power prediction: uncertainty

2.1. Short term wind power prediction

Short term wind power prediction programs are tools that provide an estimate of the future power production of a wind farm, or a group of wind farms, in the next hours. For this purpose, they use meteorological forecasts coming from a Numerical Weather Prediction (NWP) tool, and sometimes real time SCADA data from the wind farms, namely, wind power production and other values, such as measured wind speed. Data of the wind farms, such as rated power, type and availability of wind turbines, etc. are also necessary. The output of these programs is the hourly average wind farm production for the next hours. Typically, predictions are issued for the next 48 hours, but longer time horizons are possible, sometimes at the price of a poorer accuracy.

These prediction tools are less accurate than load prediction programs and their accuracy decreases with the time horizon. An example of this accuracy for a typical wind farm is given in Fig. 1, where the output from the prediction program SIPREOLICO [6] is compared to *persistence*. Persistence is a prediction method that consists of assuming that the future prediction, for the entire time horizon considered, is the current production of the wind farm.

The figure represents the Normalized Mean Average Error, defined as

$$NMAE(k) = \frac{1}{P_n} \frac{\sum_{t=1}^{N} |e(t+k/t)|}{N} \tag{1}$$

where

$$e(t+k/t) = p(t+k) - \hat{p}(t+k/t) \tag{2}$$

And $p(t+k)$ is the production of the wind farm at time $(t+k)$, while $\hat{p}(t+k/t)$ is the power predicted at time t for time $(t+k)$. P_n is the nominal power of the wind farm, and N is the number of predictions examined along the considered time. It can be seen that the wind power prediction accuracy allows for much uncertainty, and that the actual value may differ widely from the predicted one.

2.2. Uncertainty of short term wind power prediction

The predictions provided by a short term wind power prediction program are uncertain, and this uncertainty must be modeled for an adequate assessment of these predictions.

The uncertainty, and hence its probability density function, changes with the range of the wind farm power output, since this value is bounded between zero and the rated power. Besides, the power curve of a wind turbine or wind farm is nonlinear. If we assume that the wind speed forecasts have Gaussian uncertainty, then the probability density functions of the power predictions will not be Gaussian. The shape of these probability density functions is also affected by the time lag elapsed between the prediction and the operation times. A sample of an heuristical PDF of the uncertainty of short power prediction is given in Fig. 2. This function shows the uncertainty of a wind power prediction made with a time horizon of 7 hours when the forecasted power was 0.2 p.u.

It is not within the purposes of this paper to propose a model for this uncertainty and a reasonable assumption will be used as an approximation. Due to the bounded nature of the power produced by a wind far, a Beta PDF will be used, as proposed in [7]. Heuristic PDF, as shown in [8] supports this assumption, although this is still an open field for research. In our case, the mean of the distribution will be the predicted power at the time of interest, while the standard deviation σ will depend on the level of power injected, with respect to the wind farm rated power. This dependence has been obtained heuristically for some wind farms, and the results are shown in Fig. 3, where the value of standard deviation is normalized to the rated power of the wind farm. Although there are wide variations, an approximation by a quadratic curve (shown in the picture) may provide realistic results.

All these values have been obtained from real production of three months of a wind farm whose rated power has been normalized to 1. Predictions have been made with the prediction tool SIPREOLICO [6].

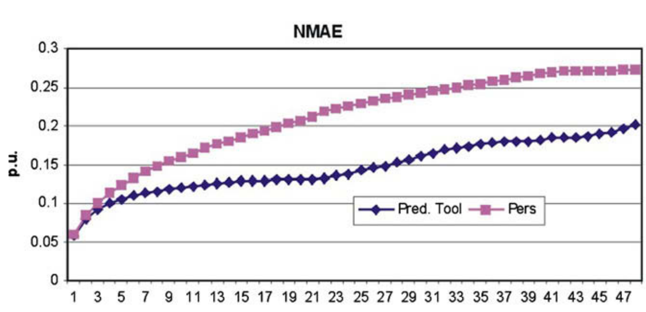

Fig. 1. NMAE of a typical wind farm, for a prediction tool and persistence.

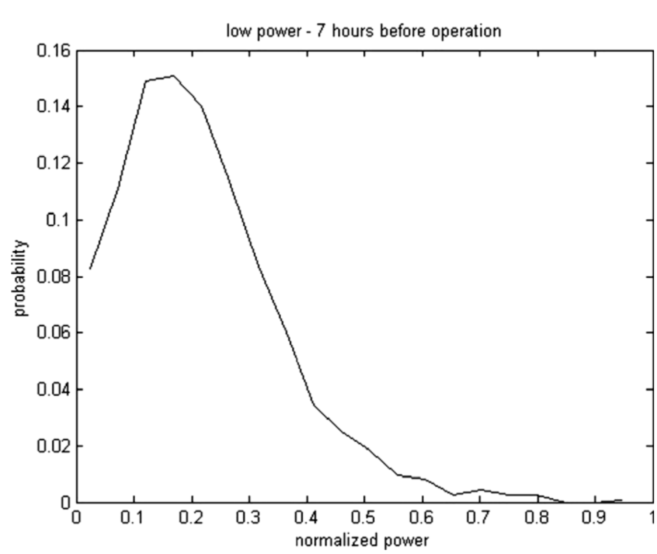

Fig. 2. Sample PDF of the uncertainty of wind power prediction.