Electric Power Systems Research 89 (2012) 129–138

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Probabilistic load flow for photovoltaic distributed generation using the Cornish–Fisher expansion

F.J. Ruiz-Rodriguez[a], J.C. Hernández[b], F. Jurado[a,*]

[a] Department of Electrical Engineering, University of Jaén, 23700 EPS Linares, Jaén, Spain
[b] Department of Electrical Engineering, University of Jaén, 23071 EPS Jaén, Spain

ARTICLE INFO

Article history:
Received 12 March 2011
Received in revised form 6 February 2012
Accepted 12 March 2012
Available online 6 April 2012

Keywords:
Irradiance
Probability density function
Distributed generation
Monte Carlo method
Probabilistic load flow

ABSTRACT

This paper shows that in order to solve a probabilistic load flow in radial distribution networks, it is necessary to apply effective techniques that take into account their technical constraints. Among these constraints, voltage regulation is one of the principal problems to be addressed in photovoltaic distributed generation. Probabilistic load flows can be solved by analytical techniques as well as the Monte Carlo method. Our research study applied an analytical method that combined the cumulant method with the Cornish–Fisher expansion to solve this problem. The Monte Carlo method is used to compare the results of analytical method proposed.

To evaluate the performance of photovoltaic distributed generation, this paper describes a probabilistic model that takes into account the random nature of solar irradiance. Therefore, load and photovoltaic distributed generation are modelled as independent/dependent random variables.

The results obtained show that the technique proposed gave a better performance than the Monte Carlo method. This technique provided satisfactory solutions with a smaller number of iterations. Therefore, convergence was rapidly attained and computational cost was lower than that required for the Monte Carlo method. Besides, the results revealed how the Cornish–Fisher expansion had a better performance than the Gram–Charlier expansion, when input random variables were non-Gaussian.

1. Introduction

Until now, the main function of distribution networks has been limited to connecting central generation and transmission networks to the end consumers. As a result, distribution networks have always been regarded as passive networks. However, the wide use of DG is in the process of transforming them and making them more active [1].

DG can affect voltage profiles since when a generator operates in a network, this causes the voltage to increase. Although, this has the advantage of making the security margin greater and of reducing losses, it can also lead to overvoltages, especially in the neighbourhood of the DG unit [2].

The connection rules and criteria for the penetration of DG are based on a deterministic steady state analysis. However, deterministic load flow analysis cannot objectively measure how often or specify the location where overvoltages or undervoltages will occur in the network over a given time period. This can be accomplished by using the probabilistic load flow based on analytical techniques or the Monte Carlo method [3]. Probabilistic load flow is described in [4,5], and is further developed in [6,7]. A new simulation method for a composite power system is proposed in [8] in order to evaluate the probability distribution function of branch flows and node voltage magnitudes.

Ref. [9] describes a study of probabilistic load flow using the cumulant method combined with the Gram–Charlier expansion to characterise the output random variables of the problem. The cumulant method exploits the properties of the convolution of random variables [10]. Another analytical technique for characterising these variables is one that combines the cumulant method with Von Mises functions [11]. Although this technique gives a better approximation than the Gram–Charlier expansion, it comes with a higher computational cost.

Despite the fact that they are somewhat less accurate, the advantage of analytical techniques, as opposed to the Monte Carlo method, is their lower computational cost.

Probabilistic techniques can also be effectively applied to analyse the optimal power flow of the systems [12,13]. The point estimate method is used in [14,15] to solve the probabilistic load flow. In Refs. [16–18], the probabilistic load flow is used in meshed power systems with wind generation.

To evaluate the performance of PV generators in large radial distribution networks, this paper proposes an analytical technique

* Corresponding author. Tel.: +34 953 648518; fax: +34 953 648586.
E-mail addresses: fjruiz@ujaen.es (F.J. Ruiz-Rodriguez), jcasa@ujaen.es (J.C. Hernández), fjurado@ujaen.es (F. Jurado).

0378-7796/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsr.2012.03.009

for probabilistic load flow based on the cumulant method combined with the Cornish–Fisher expansion. Probabilistic load flow has typically included the uncertainty of load, which is modelled with Gaussian PDFs. However, the PV energy growth poses new challenges, since the variability of PV power production is much higher, and the PDF of the uncertainties is not Gaussian [19,20]. Moreover, the uncertainties of PV power injections in geographically close PV generators are dependent on each other. As PV PDF has not as yet been satisfactorily modelled, this paper proposes a probabilistic PV model to be included in the probabilistic radial load flow. The technique proposed is an improvement over the previous approach in [21,22] because there is a substantial enhancement in the modelling of PV power production and loads. Although the time interval considered in this technique is 1 h, shorter intervals of time can be considered by varying the correlation of PV dependence and the distribution functions of load and PV power injection.

In general, the technique proposed in this paper combines different approaches. However, it is based on the cumulant method, generalised for the case of dependent random variables. Additionally, a more refined method [23] to solve the load flow in radial distribution systems is applied because the traditional Newton–Raphson method produces convergence problems in these systems.

2. Probabilistic PV system model

Solar irradiation on a horizontal surface inside the atmosphere cannot be accurately predicted since it depends on the irregular presence of clouds. The randomness produced by clouds on terrestrial irradiation is characterised by two random variables [24,25]: the daily clearness index K_T and the hourly diffuse fraction k_d.

The characterisation of the behaviour of global irradiation [24,26–30] and of diffuse irradiation [25,27,31–33] makes it possible to construct a probability model (using PDFs and CDFs) for indexes, K_T [24,27–29] and k_d [25]. Thus, the statistical properties of global irradiation are first described in terms of the daily clearness index [24,27–30]:

$$K_T = \frac{H_{g,d}}{H_{0,d}} \tag{1}$$

Hollands and Huget propose the following PDF for the random variable $\boldsymbol{K_T}$ dependent only on $\bar{K}_T$ [24]:

$$p_K(\boldsymbol{K_T}, \bar{K}_T) = C_1 \left(\frac{K_{Tu} - \boldsymbol{K_T}}{K_{Tu}}\right) e^{\lambda_1 \cdot \boldsymbol{K_T}} \tag{2}$$

where C_1 and λ_1 parameters are functions of K_{Tu} and $\bar{K}_T$.

The random variable, daily global irradiation $\boldsymbol{H_{g,d}}$, obtained from Eq. (1) is:

$$\boldsymbol{H_{g,d}} = H_{0,d} \cdot \boldsymbol{K_T} \tag{3}$$

The hourly global irradiation $\boldsymbol{H_{g,h}}$ is obtained from $\boldsymbol{H_{g,d}}$ with the Eq. (4) [34]:

$$r_g = \frac{\boldsymbol{H_{g,h}}}{\boldsymbol{H_{g,d}}} = \frac{H_{0,h}}{H_{0,d}} \cdot \theta(t) \tag{4}$$

Secondly, the statistical properties of diffuse irradiation are described in terms of the hourly diffuse fraction [25,31–33]:

$$k_d = \frac{H_{d,h}}{H_{g,h}} \tag{5}$$

Although most $k_d(k_t)$ available correlations are deterministic, an effective approach [25,31] should consider the fact that this function is not deterministic. In other words, for a given k_t, value, k_d can take a range of values distributed around its mean, $\bar{k}_d$. Using the expected value approach of probability theory, Hollands and Crha [25] gives a general-purpose expression for the PDF of the random variable $\boldsymbol{k_d}$:

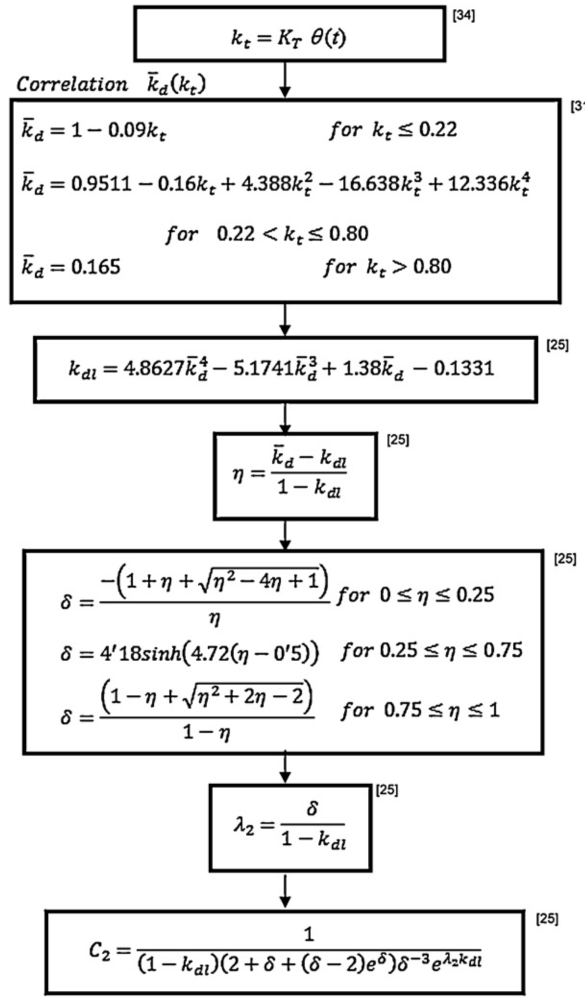

Fig. 1. Flowchart to evaluate C_2, λ_2 and k_{dl} parameters.

$$p_K(\boldsymbol{k_d}, \overline{k_d}) = C_2(\boldsymbol{k_d} - k_{dl}) \cdot (1 - \boldsymbol{k_d}) e^{\lambda_2 \cdot \boldsymbol{k_d}} \tag{6}$$

where C_2, λ_2, and k_{dl} parameters are functions of $\bar{k}_d$, as shown in Fig. 1. The expression for $p_k(\boldsymbol{k_d}, \bar{k}_d)$ is independent of the actual $\bar{k}_d(k_t)$ relation [25,31–33].

The random variable, hourly diffuse irradiation $\boldsymbol{H_{d,h}}$, which combines Eqs. (4) and (5), is given by Eq. (7):

$$\boldsymbol{H_{d,h}} = r_g \cdot H_{g,h} \cdot \boldsymbol{k_d} \tag{7}$$

The hourly global irradiance on a tilted surface G_{g,h_β} can be calculated from the components of the incident irradiance beam ($G_{b,h} = G_{g,h} - G_{d,h}$), diffuse irradiance ($G_{d,h}$), and ground-reflected irradiance ($G_{r,h}$) on the horizontal plane [35]:

$$\begin{aligned} G_{g,h\_\beta} &= (G_{g,h} - G_{d,h}) R_b FT_b(\theta_S) \\ &+ G_{d,h}\left[(1-\bar{k}_b)\frac{1+\cos\beta}{2} FT_d(\beta) + R_b \bar{k}_b FT_b(\theta_S)\right] \\ &+ G_{g,h}\rho\left(\frac{1-\cos\beta}{2}\right) FT_r(\beta) \end{aligned} \tag{8}$$

To determine the diffuse irradiance fraction, the Hay model [36] for anisotropic skies is applied in Eq. (8). Furthermore, the angular losses of each radiation component, $FT_b(\theta_s)$, $FT_d(\beta)$, and $FT_r(\beta)$, follow Ref. [35]. The global, beam, and diffuse irradiance are assumed