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Cost minimization of measurement devices under estimation accuracy constraints in the presence of Gaussian noise ☆

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ABSTRACT

Novel convex measurement cost minimization problems are proposed based on various estimation accuracy constraints for a linear system subject to additive Gaussian noise. Closed form solutions are obtained in the case of an invertible system matrix. In addition, the effects of system matrix uncertainty are studied both from a generic perspective and by employing a specific uncertainty model. The results are extended to the Bayesian estimation framework by treating the unknown parameters as Gaussian distributed random variables. Numerical examples are presented to discuss the theoretical results in detail.

1. Introduction

In this paper, we propose measurement cost minimization problems under various constraints on estimation accuracy for a system characterized by a linear input–output relationship subject to Gaussian noise. For the measurement cost, we employ the recently proposed measurement device model in [1], and present a detailed treatment of the proposed measurement cost minimization problems. Although the statistical estimation problem in the presence of Gaussian noise is by far the most widely known and well-studied subject of estimation theory [2], approaches that consider the estimation performance jointly with system-resource constraints have become popular in recent years. Distributed detection and estimation problems took the first step by incorporating bandwidth and energy constraints due to data processing at the sensor nodes, and data transmission from sensor nodes to a fusion node in the context of wireless sensor networks (WSNs) [3–7]. Since then, the majority of the related studies have addressed the costs arising from similar system-level limitations with a relatively weak emphasis on the measurement costs due to amplitude resolution and dynamic range of the sensing apparatus. To begin with, we summarize the main aspects of the research that has been carried out in recent years to unfold the relationship between estimation capabilities and aforementioned costs of the sensing devices.

In [3], detection problems are examined under a constraint on the expected cost resulting from measurement and transmission stages. It is found out that optimal detection performance can be achieved by a randomized on–off transmission scheme of the acquired measurements at a suitable rate. The distributed mean-location parameter estimation problem is considered in [4] for WSNs based on quantized observations. It is shown that when the dynamic range of the estimated parameter is small or comparable with the noise variance, a class of maximum likelihood (ML) estimators exists with performance close to that of the sample mean estimator under stringent bandwidth constraint of one bit per sensor. When the dynamic range of the estimated parameter is comparable to or large than the noise variance, an optimum value for the quantization step results in the highest estimation accuracy possible for a given bandwidth constraint. In [5], a power scheduling strategy that minimizes the total energy consumption subject to a constraint on the worst mean-squared-error (MSE) distortion is derived for decentralized estimation in a heterogeneous sensing environment. Assuming an uncoded quadrature amplitude modulation (QAM) transmission scheme and uniform randomized quantization at the sensor nodes, it is stated that depending on the corresponding channel quality, a sensor is either on or off completely. When a sensor is active, the optimal values for transmission power and quantization level for the sensor can be determined analytically in terms of the channel path losses and local observation noise levels.

In [6], distributed estimation of an unknown parameter is discussed for the case of independent additive observation noises with possibly different variances at the sensors and over non-ideal fading wireless channels between the sensors and the fusion center. The concepts of estimation outage and estimation diversity are introduced. It is proven that the MSE distortion can be minimized under sum power constraints by turning off sensors transmitting over bad channels adaptively without degrading the diversity gain. In addition, performance decrease is reported when

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individual power constraints are also imposed at each sensor. In [7], the distributed estimation of a deterministic parameter immersed in uncorrelated noise in a WSN is targeted under a total bit rate constraint. The number of active sensors is determined together with the quantization bit rate of each active sensor in order to minimize the MSE. The problem of estimating a spatially distributed, time-varying random field from noisy measurements collected by a WSN is investigated under bandwidth and energy constraints on the sensors in [8]. Using graph-theoretic techniques, it is shown that the energy consumption can be reduced by constructing reduced order Kalman–Bucy filters from only a subset of the sensors. In order to prevent degradation in the root-mean-squared (RMS) estimation error performance, efficient methods employing Pareto optimality criterion between the communication costs and RMS estimation error are presented. A power allocation problem for distributed parameter estimation is investigated under a total network power constraint for various topologies in [9]. It is shown that for the basic star topology, the optimal solution assumes either of the sensor selection, water-filling, or channel inversion forms depending on the measurement noise variance, and the corresponding analytical expressions are obtained. Asymptotically optimal power allocation strategies are derived for more complex branch, tree, and linear topologies assuming amplify-and-forward and estimate-and-forward transmission protocols. The decentralized WSN estimation is extended to incorporate the effects of imperfect data transmission from sensors to fusion center under stringent bandwidth constraints in [10].

Important results are also obtained for the sensor selection problem under various constraints on the system cost and estimation accuracy. The problem of choosing a set of k sensor measurements from a set of m available measurements so that the estimation error is minimized is addressed in [11] under a Gaussian assumption. It is shown that the combinatorial complexity of the solution can significantly be reduced without sacrificing much from the estimation accuracy by employing a heuristic based on convex optimization. In [12], a similar sensor selection problem is analyzed in a target detection framework when several classes of binary sensors with different discrimination performance and costs are available. Based on the conditional distributions of the observations at the fusion center, the performance of the corresponding optimal hypothesis tests is assessed using the symmetric Kullback–Leibler divergence. The solution of the resulting constrained maximization problem indicates that the sensor class with the best performance-to-cost ratio should be selected.

As outlined above, not much work has been performed, to the best of our knowledge, in the context of jointly designing the measurement stage from a cost-oriented perspective while performing estimation up to a predetermined level of accuracy. In other words, the trade-offs between measurement associated costs and estimation errors remain, to a large extent, undiscovered in the literature. On the other hand, if adopted, such an approach will inevitably require a general and reliable method of assessing the cost of measurements applicable to any real world phenomenon under consideration as well as an appropriate means of evaluating the best achievable estimation performance without reference to any specific estimator structure. For the fulfillment of the first requirement, a novel measurement device model is suggested in [1], where the cost of each measurement is determined by the number of amplitude levels that can reliably be distinguished. As a consequence, higher resolution (less noisy) measurements demand higher costs in accordance with the usual practice. Although the proposed model may lack in capturing the exact relationship between the cost and inner workings of any specific measurement hardware, it encompasses a sufficient amount of generality to remain useful under a multitude of circumstances. Based on this measurement model, an optimization problem is formulated in [13] in order to calculate the optimal costs of measurement devices that maximize the average Fisher information for a scalar parameter estimation problem.

Although the optimal cost allocation problem is studied for the single parameter estimation case in [13], and the signal recovery based on linear minimum mean-squared-error (LMMSE) estimators is discussed under cost-constrained measurements using a linear system model in [1], no studies have analyzed the implications of the proposed measurement device model in a more general setting by considering both random and nonrandom parameter estimation under various estimation accuracy constraints and uncertainty in the linear system model. The main contributions of our study in this paper extend far beyond a multivariate analysis of the discussion in [13], and can be summarized as follows:

- Formulated new convex optimization problems for the minimization of the total measurement cost by employing constraints on various estimation accuracy criteria (i.e., different functionals of the eigenvalues of the Fisher information matrix (FIM)) assuming a linear system model[1] in the presence of Gaussian noise.
- Studied system matrix uncertainty both from a general perspective and by employing a specific uncertainty model.
- Obtained closed form solutions for two of the proposed convex optimization problems in the case of invertible system matrix.
- Extended the results to the Bayesian estimation framework by treating the unknown estimated parameters as Gaussian distributed random variables.

In addition to the items listed above, simulation results are presented to discuss the theoretical results. Namely, we compare the performance of various estimation quality metrics through numerical examples using optimal and suboptimal cost allocation schemes, and simulate the effects of system matrix uncertainty. We also examine the behavior of the optimal solutions returned by various estimation accuracy criteria under scaling of the system noise variances, and identify the most robust criterion to variations in the average system noise power via numerical examples. The relationship between the number of effective measurements and the quality of estimation is also investigated under scaling of the system noise variances.

The rest of this paper is organized as follows: In Section 2, we pose the optimal cost allocation problem as a convex optimization problem under various information criteria for nonrandom parameter vector estimation. In Section 3, we modify the proposed optimization problems to handle the worst-case scenarios under system matrix uncertainty. Next, we take a specific but nevertheless practical uncertainty model, and discuss how the optimization problems are altered while preserving convexity. In Section 4, we focus on two optimization problems proposed in Section 2, and simplify them to obtain closed form solutions in the case of invertible system matrix. In Section 5, we provide several numerical examples to illustrate the results presented in this paper. Extensions to Bayesian estimation with Gaussian priors are discussed in Section 6, and we conclude in Section 7.

2. Optimal cost allocation under estimation accuracy constraints

Consider a discrete-time system model as in Fig. 1 in which noisy measurements are obtained at the output of a linear system, and then the measurements are processed to estimate the value of a nonrandom parameter vector $\boldsymbol{\theta}$. The observation vector $\mathbf{x}$ at the

[1] Such linear models have a multitude of application areas, a few examples of which are channel equalization, wave propagation, compressed sensing, and Wiener filtering problems [14,15].