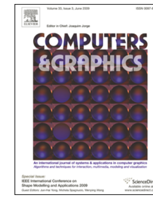
Computers & Graphics 47 (2015) 145–164

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag



Survey Paper

Architectural geometry

Helmut Pottmann [a,b], Michael Eigensatz [c], Amir Vaxman [b], Johannes Wallner [d,*]

[a] King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia
[b] Vienna University of Technology, 1040 Vienna, Austria
[c] Evolute GmbH, 1040 Vienna, Austria
[d] Graz University of Technology, 8010 Graz, Austria

ARTICLE INFO

Article history:
Received 7 July 2014
Received in revised form
20 October 2014
Accepted 17 November 2014
Available online 26 November 2014

Keywords:
Discrete differential geometry
Fabrication-aware design
Paneling
Support structures
Polyhedral patterns
Statics-aware design

ABSTRACT

Around 2005 it became apparent in the geometry processing community that freeform architecture contains many problems of a geometric nature to be solved, and many opportunities for optimization which however require geometric understanding. This area of research, which has been called architectural geometry, meanwhile contains a great wealth of individual contributions which are relevant in various fields. For mathematicians, the relation to discrete differential geometry is significant, in particular the integrable system viewpoint. Besides, new application contexts have become available for quite some old-established concepts. Regarding graphics and geometry processing, architectural geometry yields interesting new questions but also new objects, e.g. replacing meshes by other combinatorial arrangements. Numerical optimization plays a major role but in itself would be powerless without geometric understanding. Summing up, architectural geometry has become a rewarding field of study. We here survey the main directions which have been pursued, we show real projects where geometric considerations have played a role, and we outline open problems which we think are significant for the future development of both theory and practice of architectural geometry.

1. Introduction

Free forms constitute one of the major trends within contemporary architecture. In its earlier days a particularly important figure was Frank Gehry, with his design approach based on digital reconstruction of physical models, resulting in shapes which are not too far away from developable surfaces and thus ideally suited for his preferred characteristic metal cladding [94]. Nowadays we see an increasing number of landmark buildings involving geometrically complex freeform skins and structures (Fig. 1).

While the modeling of freeform geometry with current tools is well understood, the actual fabrication on the architectural scale is a challenge. One has to decompose the skins into manufacturable panels, provide appropriate support structures, meet structural constraints and last, but not least make sure that the cost does not become excessive. Many of these practically highly important problems are actually of a geometric nature and thus the architectural application attracted the attention of the geometric modeling and geometry processing community. This research area is now called *Architectural Geometry*. It is the purpose of the present survey to provide an overview of this field from the Computer Graphics perspective. We are not addressing here the many beautiful designs which have been realized by engineers with a clever way of using state of the art software, but we are focusing on research contributions which go well beyond the use of standard tools. This research direction has also been inspired by the work of the smart geometry group (www.smartgeometry.com), which promoted the use of parametric design and scripting for mastering geometric complexity in architecture.

From a methodology perspective, it turned out that the probably two most important ingredients for the solution of Architectural Geometry problems are *Discrete Differential Geometry* (DDG) [16,84] and *Numerical Optimization*. In order to keep this survey well within Graphics, we will be rather short in discussing the subject from the DDG perspective and only mention those insights which are essential for a successful implementation. It is a fact that understanding a problem from the DDG viewpoint is often equivalent to understanding how to successfully initialize and solve the numerical optimization problems which are more directly related to the questions at hand.

In general, the approximation of an ideal design surface by a surface which is suitable for fabrication is called *rationalization* in Architecture. This often means *panelization*, i.e. finding a collection of smaller elements covering the design surface, but it can also mean replacing the design surface by a surface which has a simple generation like a ruled surface. Often, rationalization is harder than the 3D modeling of a surface. A digital modeling tool which automatically generates only buildable structures of a certain type (*fabrication-aware design*) is probably more efficient than the still

* Corresponding author.
E-mail address: j.wallner@tugraz.at (J. Wallner).

http://dx.doi.org/10.1016/j.cag.2014.11.002
0097-8493/

Fig. 1. Complex architecture entails a complex workflow. This image shows part of the Fondation Louis Vuitton, Paris, designed by F. Gehry. (Photo: Mairie de Paris).

prevalent approach based on rationalization. For research, both rationalization and fabrication-aware design are interesting, but the latter poses more unsolved problems, at the same time going far beyond architecture.

The solution of the above-mentioned problems may become easier if the shape under consideration has special properties, in which case we do not call it truly freeform. E.g. a surface generated by translation is easily rationalized into flat quadrilaterals (see Fig. 6). Special shapes have been extensively and very successfully employed, but this paper focuses on properties and algorithms relating to arbitrary (freeform) shapes.

Remark. The reader is advised that we use the word *design* in its purely technical sense, meaning that a designer uses available tools (drawings, software) to convert ideas to a geometric representation. We never refer to those aspects of design which touch cognitive science or artificial intelligence.

1.1. Overview of the paper

This paper is a survey, discussing a wide range of topics. It is divided into sections as follows: Section 2: *Flat panels*; Section 3: *Developable panels*; Section 4: *Smooth double-curved skins*; Section 5: *Paneling*; Section 6: *Support structures*; Section 7: *Repetition*; Section 8: *Patterns*; Section 9: *Statics*; Section 10: *Shading and other functional aspects*; Section 11: *Design exploration*. Within each section we address the following points:

- We point out *why* a certain topic gets addressed and which practical aspects are motivating it.
- We discuss the most essential and interesting aspects of the methodology for its solution.
- Results are provided along with a discussion which is based on real projects wherever possible.
- We address open problems and directions of future research.

We tried to make this survey as self-contained as possible. However, some background on geometry processing, optimization and elementary differential geometry is necessary. For a very simple presentation of the geometry background along with a path towards ongoing research in Architectural Geometry, we refer to [77]. For collections of publications related to the field, we especially point to the volumes [11,21,43].

2. Polyhedral surfaces – structures from flat panels

In order to realize a freeform surface in architecture, one often breaks it into smaller elements, called *panels*. Certainly, flat panels are the easiest and cheapest to produce and thus surfaces

Fig. 2. Triangle meshes in freeform architecture. Laying out a triangular pattern on a freeform surface and controlling its density and flow is a challenging problem successfully solved in 2000 by Chris Williams for the Great Court Roof of the British Museum in London (designed by Foster + Partners). Here dynamic relaxation was employed to aesthetically distribute the triangles (Photo: Waagner-Biro Stahlbau).

composed of flat panels, the so-called *polyhedral surfaces* or *polyhedral meshes* play a key role in Architectural Geometry. In this section, we discuss their various types, with a focus on meshes from planar quads. They have turned out to be the most interesting species of panel from the viewpoint of mathematics.

2.1. History of polyhedral surfaces in architecture

Until the 20th century, polyhedral surfaces in architecture appeared predominately as rotational surfaces used for roof and dome structures. Quads were the base polygon of choice for most of these early examples, mainly due to their aesthetic qualities and their economic advantages.

In 1928, engineers of the Carl Zeiss optical company used a triangulated spherical dome structure for a planetarium in Jena, Germany. The triangulation was derived by projecting the vertices of a platonic solid with regularly triangulated faces to a sphere. 20 years later, R. Buckminster Fuller reinvented, developed and popularized this concept under the name of “geodesic dome”. The increased stability of triangular elements compared to quads was one of the advantages of geodesic domes.

Later, Computer-Aided Design, by augmenting and replacing the more traditional tools for modeling freeform geometry, not only enabled more complex structures, but even created a demand for them. Designers suddenly had significantly more powerful design tools at their disposal, which in turn drove engineers to meet the challenge and again provide means of cost-effective production. Last, but not least, the gap between design freedom and production was the main driving force for mathematicians and computer scientists to found the interdisciplinary research field of Architectural Geometry.

We discuss the newer developments regarding meshes and free forms below, starting with triangle meshes.

2.2. Triangle meshes from the architectural perspective

Triangle meshes are ubiquitously used in Graphics. Their use in architecture has different reasons and different aspects become important. We list some of them:

- Edges are often visible and smoothness (or lack of smoothness) of mesh polylines is an important part of the design. This is in contrast with geometry processing and computer graphics, where triangle meshes serve as amorphous representations of shapes and the orientation of edges hardly plays a role.
- The same is true for the placement of singularities (non-valence 6 vertices). The combinatorics (i.e. connectivity) is to