ACADEMIC
PRESS

On the pricing of real estate index linked swaps

Tomas Björk* and Eric Clapham

Department of Finance, Stockholm School of Economics, Box 6501, S-113 83 Stockholm, Sweden

Abstract

In this paper we discuss the pricing of commercial real estate index linked swaps (CREILS). This particular pricing problem has been studied by Buttimer et al. in a previous paper in this journal (6 [1997]: 16). We show that their results are only approximately correct and that the true theoretical price of the swap is in fact equal to zero. This result is shown to hold regardless of the specific model chosen for the index process, the dividend process, and the interest rate term structure. We provide an intuitive economic argument as well as a full mathematical proof of our results. In particular we show that the nonzero result in the previous paper is due to two specific numerical approximations introduced in that paper, and we discuss these approximation errors from a theoretical as well as from a numerical point of view.
© 2002 Elsevier Science (USA). All rights reserved.

1. Introduction

The object under study in the present paper is a commercial real estate index linked swap (CREILS). The basic construction of such a swap is that

---

*Corresponding author. Fax: +46-8-31-23-27.
E-mail addresses: fintb@hhs.se (T. Björk), finec@hhs.se (E. Clapham).

the appreciation and yield of a given real estate index is swapped, quarterly, against the three months spot LIBOR. In an interesting paper previously published in this journal, Buttimer et al. (1997) presented a two-state model for pricing securities dependent upon a real estate index as well as upon an interest rate, and the model was then used to calculate the arbitrage-free value of a CREILS. For this concrete application, the authors in Buttimer et al. (1997) used a numerical method based upon replacing their original continuous time model by a bivariate binomial tree, and it was found that, for a notional amount of \$10,000,000, the value (to the receiver of the swap) was around \$50. Buttimer et al. (1997) then proceed to discuss the sensitivity of their numerical results to changes in volatilities, correlations and the initial term structure.

The object of the present paper is to show that the results in Buttimer et al. (1997) are only approximatively true, in the sense that the arbitrage-free theoretical value of the CREILS is in fact exactly equal to zero. More precisely we carry out the following program.

- In Section 2 we present the institutional setup of the swap.
- We begin the theoretical analysis in Section 3 where we give a simple verbal arbitrage argument showing that the theoretical value of the swap in fact equals zero.
- In Section 4 we add to the verbal discussion in the previous section by presenting a very general mathematical framework for the swap along the following lines.
  - The real estate index is allowed to be a general (semimartingale) process with the only requirement that it should be possible to view it as the price of a traded asset.
  - The income (divident) process associated to the index is allowed to be completely general.
  - The interest rate model is allowed to be completely general.
  - We assume absence of arbitrage.

This framework is considerably more general than that of Buttimer et al. (1997) where the index is assumed to be lognormal with a constant dividend yield, and where the interest rate structure is given by a CIR short rate model.

- Within the above framework, and using the standard (martingale) machinery of arbitrage theory, we prove formally that the arbitrage-free value of the swap is exactly equal to zero.
- In Section 5 we discuss why the pricing results in Buttimer et al. (1997) differ (although not much) from the correct value zero. We show that the reasons for the nonzero computational results in Buttimer et al. (1997) are due to two specific approximation errors introduced in the numerical calculations. We discuss these errors from a theoretical as well as a numerical perspective.