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Graph invariants for unique localizability in cooperative localization of wireless sensor networks: Rigidity index and redundancy index☆

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ABSTRACT

Rigidity theory enables us to specify the conditions of unique localizability in the cooperative localization problem of wireless sensor networks. This paper presents a combinatorial rigidity approach to measure (i) generic rigidity and (ii) generalized redundant rigidity properties of graph structures through graph invariants for the localization problem in wireless sensor networks. We define the rigidity index as a graph invariant based on independent set of edges. It has a value between 0 and 1, and it indicates how close we are to rigidity. Redundant rigidity is required for global rigidity, which is associated with unique realization of graphs. Moreover, redundant rigidity also provides rigidity robustness in networked systems against structural changes, such as link losses. Here, we give a broader definition of redundant edge that we call the "generalized redundant edge." This definition of redundancy is valid for both rigid and non-rigid graphs. Next, we define the redundancy index as a graph invariant based on generalized redundant edges. It also has a value between 0 and 1, and it indicates the percentage of redundancy in a graph. These two indices allow us to explore the transition from non-rigidity to rigidity and the transition from rigidity to redundant rigidity. Examples on graphs are provided to demonstrate this approach. From a sensor network point of view, these two indices enable us to evaluate the effects of sensing radii of sensors on the rigidity properties of networks, which in turn, allow us to examine the localizability of sensor networks. We evaluate the required changes in sensing radii for localizability by means of the rigidity index and the redundancy index using random geometric graphs and clustered graphs in simulations.

1. Introduction

Localization is an essential service for many applications of wireless sensor networks. A wireless sensor network consists of a small number of anchors (reference nodes) and a large number of small, cheap ordinary nodes (non-anchors). Anchors have a priori knowledge of their own positions, e.g., GPS, and ordinary nodes have no prior knowledge of their locations. If ordinary nodes were capable making measurements to multiple anchors, they could determine their positions. However, several ordinary nodes cannot directly communicate with anchors because of power limitations or signal blockage. One feature of wireless sensor networks is the cooperative effort of sensor nodes [1]. A recent paradigm is cooperative localization, in which ordinary nodes help each other to determine their locations [2,3]. In cooperative localization, ordinary nodes not only make measurements with anchors, but also they

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make measurements with other ordinary nodes. The types of measurements usually include distance estimates and/or angle estimates [4].

The set of ordinary nodes is uniquely localizable if there is a unique set of positions satisfying the conditions resulting from measurements. Note that this candidate set of positions is subject to the trivial degrees of freedom in d-space ($d = 2, 3$). For example, in 2-space, a framework with at least two vertices always has three trivial degrees of freedom generated by two translations and one rotation. Unique solvability of the cooperative localization problem is characterized by the results from "rigidity theory." Redundant rigidity is required for global rigidity, which is associated with unique realization of graphs. Use of rigidity theory in localization is well described in the literature [5–11]. More details will be given in Section 2.

Recent works introducing the rigidity theory into formation control has also provided provably correct methods to model and analyze the ad hoc network topologies within robotic teams [12–16]. For example, rigidity theory provides us tools for formation control using relative distance measurements instead of relative position measurements. Moreover, rigidity is necessary to estimate relative positions using only relative distance measurements [12].

Quantitative measures of rigidity have been proposed by researchers recently. Jacobs et al. [17] provided a measure of rigidity within the context of microstructures of proteins, and their approach is based on chemical bonds. Zhu and Hu [18] studied quantitative measure of formation rigidity using stiffness matrix. Zelazo et al. [12] introduced the rigidity eigenvalue based on symmetric rigidity matrix. The latter two studies employed rigidity based matrices and studied the properties of those matrices to explore the rigidity properties of networks. Preliminary results on measures of redundant rigidity based on only rigid graphs, without consideration of non-rigid graphs, were provided in [19].

In this paper, first we provide a measure of "generic rigidity" for both rigid and non-rigid graphs. Then the concept of generalized redundancy is introduced, which allows us to provide a measure of "generalized redundancy" for both rigid and non-rigid graphs. Our approach is based on the combinatorial characterizations of rigidity and redundancy. Specifically, the main contributions of this work are: (a) the translation of edge distribution in a network graph to that of a rigidity measure that we term the "rigidity index," (b) the translation of the generalized redundancy of edge distribution in a network graph to that of a redundancy measure on network rigidity that we term the "redundancy index."

From a graph theory point of view, the benefits of these measures are as follows: they permit us (i) to quantify the distribution of edges in a graph in terms of rigidity and redundant rigidity, (ii) to compare various graphs for rigidity in terms of the distribution of their redundant and non-redundant edges.

From a sensor network point of view, these two measures enable us to evaluate the effects of sensing radii of sensors on the rigidity and redundancy properties of networks, which in turn allows us to examine the localizability of sensor network graphs. In particular, we are interested in the following questions: (i) how much change in sensing radii do we need to reach from non-rigidity to rigidity, and to reach from rigidity to redundant rigidity in random geometric graphs? (ii) Given that redundant rigidity is associated with unique localizability, is redundant rigidity a heavy burden on the network once rigidity is achieved? We provide answers to these questions in this paper.

The localization process often needs to be repeated in mobile wireless sensor networks. Mobility brings the possibility of the loss of links, which enforces to have not only localizable network structures but also structures which remain localizable after the loss of links in the network. Since redundancy plays a role in robustness, redundancy measure also helps us to evaluate robustness to link losses.

The structure of the paper is as follows. We give preliminaries on rigidity in Section 2. Main results on the rigidity index, the redundancy index and the corresponding complexity analysis are provided in Section 3. Examples to illustrate those indices on graphs are presented in Section 4. Applications of these two indices in sensor network simulations are demonstrated in Section 5. Finally, the paper ends with a conclusion and some outlook on future directions in Section 6.

2. Rigidity

First we provide below some background on rigidity, redundant rigidity and global rigidity. We refer the reader to [20–23] and the references therein for more details.

2.1. Rigid frameworks and the rigidity matrix

We model a network by a finite graph $G = (V, E)$. All graphs considered are finite without loops and multiple edges. Nodes of the network correspond to the vertices of G, and for every link in the network there is an edge joining the corresponding vertices of the graph. A *framework* $G(p)$ is a graph $G = (V, E)$ and a plane configuration p: $V \to R^2$. Two frameworks $G(p)$ and $G(q)$ are *equivalent* if $||p(v_i) - p(v_j)|| = ||q(v_i) - q(v_j)||$ holds whenever v_iv_j corresponds to an edge of G, where $\|.\|$ denotes the distance. $G(p)$ and $G(q)$ are *congruent* if for any two vertices v_i, $v_j \in V$, $||p(v_i) - p(v_j)|| = ||q(v_i) - q(v_j)||$ holds. A framework $G(p)$ in R^2 is *rigid* if there is an $\epsilon > 0$ such that for any other configuration q in R^2, where $||p(v) - q(v)|| < \epsilon$ for all v in V and $G(p)$ is equivalent to $G(q)$, then p is congruent to q. Intuitively, we may consider the rigidity of bar-joint frameworks. Here, bars correspond to edges, and joints correspond to vertices. A bar-joint framework is rigid if it has only trivial deformations, e.g., translations and rotations.

The *rigidity matrix* $R(G, p)$ of a framework $G(p)$ is the $|E| \times 2|V|$ matrix, whose rows correspond to the edges and whose columns correspond to the coordinates of the vertices, where $|.|$ denotes the cardinality of a set. If $e = v_iv_j \in E$, then the entry in the row e and the column v_i is $p(v_i) - p(v_j)$, the entry in the row e and the column v_j is $p(v_j) - p(v_i)$, and the other entries in the row e are zeros. If $e = v_iv_j$ is not in E, then the entire row e is zeros. A framework (G, p) is called *infinitesimally rigid* if