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Towards logical operations research—propositional case

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Abstract

Tautology is interpreted as a necessary condition for the workability of an operations system. This condition suggests the following possibilities: the stable solvability of balance equations between available resources and requests for them; the calculation of potential and kinetic of the system together with the estimation of the contribution of every operation to the kinetic of the system; the construction of a deadlockless infinite cyclic process for performance of some works.

Keywords: Tautology; Solvability of linear systems; Dynamic system

1. Introduction

The semantics of propositional calculus suitable for the simulation of operations research will be presented.

The usual assignment of *true* or *false* values for literals has made it possible to interpret the set of all deducible in propositional calculus as the set of all tautologies. Thus, the simplest stratum of the capacious notion of truth is formalized. We can suggest the essentially different interpretations of the set of deducible formulas, and we can show that predicate calculus can be considered as the formalization of other, yet more interesting interpretations.

For the sake of definiteness, we use the DNF case. Let every literal be assigned a positive real number in such a way that the sum of these numbers is equal to 1 for every propositional variable. Then the literals can be associated with elementary random events, and a conjunction of the literals is interpreted as an event when all corresponding elementary events of constituting literals occur jointly. The disjunction

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of conjunctions is interpreted as a composite event when at least one from the events associated with conjunctions occurs. It turns out that the DNF formula is tautological iff the associated event is certain [3, 8].

On the one hand, this semantics gives evident application of logic to probability theory. On the other hand, it allows to decide whether a formula is tautological by the standard practice for the calculation of the composite event probability: the "inclusing–exclusing" principle and the use of the calculation of the complementary event probability, etc. By this technique a new class of formulas with polynomial calculation of such probability can be defined [9].

Another example is the interpretation of the DNF formula by a system of linear homogeneous algebraic equations. In this case, every propositional variable is associated with an equation, in which every positive (negative) occurence of this propositional variable in a DNF formula is presented by positive (negative) monomial. Further, all monomials, corresponding with the literal occurrences in one conjunction, contain the same algebraic variable. It turns out that a formula is tautological iff the associated system has non-zero non-negative solution for any monomial coefficients [4].

Hence tautological DNF is taking a meaning of stable solvable system of balance equations in which every equation describes income and expenditure of some resource, and every algebraic variable describes the joint variation of different resources. Moreover, for every solvable system $Ax = 0,\ x \geqslant 0$, there exists such stable solvable system $Bx = 0,\ x \geqslant 0$, that every equation of system $Ax = 0$ is a linear combination of equations of system $Bx = 0$ [5]. In particular, this semantics allows to study possible variations of coefficients for which a homogeneous system preserves its non-zero non-negative solvability.

In this paper basic concepts of the two above-mentioned interpretations are uniting in a new semantics of *logistic process*. One step of a logistic process (a production process, a transport process and so on) can be represented as the performance of some elementary work (or operation) for which all the required conditions are ripe. A cyclic sequence of such steps can be interpreted as the stable logistic chain or the work repeated many times. *Tautological DNF began to take on a meaning of an operations system in which a stable logistic process exists.*

In more detail, on the one hand, we assume that propositional variables describe a life of turning resources in such a way that one propositional variable is associated with one resource. The number of different phases of a resource is equal to the number of different literals of the associated propositional variable (it is equal to 2 in the case of two-valued variables). It is supposed that every unit of one resource makes a complete turn, i.e. a cyclic round of its phases. Moreover, it is supposed that every phase of a resource unit is provided with a positive number—a potential which is forcing to move the unit of a resource into the next phase. This potential is realized when the phase is demanded.

On the other hand, we assume that a DNF formula describes the possible movements among turning resources in the following way. Let every literal from a conjunction be assigned a positive number. We interpret the conjunction of literals with their numbers