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Analyzing the Potential of a Firm: An Operations Research Approach

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Abstract—An approach to analyzing the potential of a firm, which is understood as the firm's ability to provide goods or (and) services to be supplied to a marketplace under restrictions imposed by a business environment in which the firm functions, is proposed. The approach is based on using linear inequalities and, generally, mixed variables in modelling this ability for a broad spectrum of industrial, transportation, agricultural, and other types of firms and allows one to formulate problems of analyzing the potential of a firm as linear programming problems or mixed programming problems with linear constraints. This approach generalizes the one proposed by the author earlier for a more narrow class of models and allows one to effectively employ a widely available software for solving practical problems of the considered kind, especially for firms described by large scale models of mathematical programming.

Keywords—Compatible systems of linear inequalities, Linear programming, Mixed programming problems.

1. INTRODUCTION

The potential of a firm is usually understood as its ability to provide products (goods or services) to be supplied to a marketplace under restrictions imposed by a business environment in which the firm functions. This business environment is determined by technologies employed or available to be employed, equipment in use or available for using, the firm's labor forces, management team, available resources, etc., and it is obvious that there exist a broad variety of kinds of businesses for which the business environment is different. Although for some firms, their potential is fully determined by their financial abilities and qualification of the personnel (for instance, consulting firms), for industrial, transportation, agricultural, and a variety of other kinds of firms, even finding whether or not certain products (for instance, goods or services that can be technologically produced) could be provided in particular volumes presents considerable difficulties not saying about any formalized description of sets of such feasible volumes.

For instance, for a refinery enterprise producing gasoline of several kinds, diesel fuel, black oil, as well as other products, from crude oil and having a number of equipment units each of which may function under various technological conditions, finding whether or not a certain amount of a particular type of gasoline can be produced from a particular amount of crude oil, along with other products to be produced in certain amounts, presents substantial difficulties even for specialists in refinery industry in many (nontrivial) cases [1]. The same is true for transportation systems, for instance, for certain container lines operating several vessels calling on several ports in one

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or several regions in finding whether or not a particular cargo should be taken in a particular direction proceeding from cargo flows, tariffs for transporting containers with each type of cargo, as well as those for transporting empty containers, amounts of empty containers concentrated at certain ports to which this cargo is to be moved, and expenses associated with leasing containers and storing empty containers in the ports [2].

Employing mathematical modelling and optimization methods, first of all, linear programming methods has proved effective for the analysis of the potential of numerous firms, and approaches to the analysis were proposed, in particular, by the author for systems whose mathematical models can be represented by systems of linear inequalities of the kind

$$\begin{aligned} A_1 x &\geq b_1, \qquad x \in R_+^n, \\ A_2 x &\leq b_2, \\ A_3 x &\leq b_3, \end{aligned} \tag{1}$$

in which the system $A_1 x \geq b_1$ reflects relations existing in a particular firm that are associated with producing products and (or) providing services in volumes determined by components of the vector b_1, the system $A_2 x \leq b_2$ in which each column of the matrix A_2 has at least one nonzero element reflects relations pertaining to the equipment and technologies employed, whereas the system $A_3 x \leq b_3$ reflects relations attributed to resources consumed by the firm whose volumes are determined by components of the vector b_3 [3]. In all these models, elements of matrices A_1, A_2, A_3, as well as components of the vector x having corresponding dimensions, are considered real members. Some examples of models having such a structure, along with meaningful descriptions of elements of the matrices, can be found, in particular, in [4]–[6].

The author's more than 30 years experience in the mathematical modelling of industrial, transportation, agricultural, and servicing firms has shown that models of an overwhelming majority of them can be described by systems of the kind (1) with, possibly, additional restrictions of the kind

$$\langle e_i, x \rangle \in N, \qquad i \in I \subset \overline{1,n}, \tag{2}$$

where e_i is the vector all components which equal 0, except for component $i \in \overline{1,n}$, of which equals 1, and N is the set of all natural numbers. The purpose of this article is to present an approach to the analysis of the potential of firms whose mathematical models can be described by relations (1),(2), along with examples illustrating the use of mathematical models of the kind (1),(2). Some of the considerations presented are based on the author's results that have been published in Russian in collections of articles that are hard to get access to.

2. EXAMPLES OF MATHEMATICAL MODELS OF THE KIND (1),(2)

To back up the assertion on a wide proliferation of mathematical models of the kind (1),(2), made in the introduction, three mathematical models of the kind (1) taken from scientific publications and pertaining to industrial, transportation, and agricultural systems, which were presented by the author in [7], are reproduced in this section, and a modification of one of the models which represents a model of the kind (1),(2) is also used to this end. A special nonlinear model describing regularities pertaining to the vocational training and retraining of employees of an enterprise, which is reducible to the model of the kind (1),(2), was proposed by the author in [8]. However, this model seems too cumbersome to be reproduced in this article.

A machine-building firm produces n products using m groups of equipment, L types of row materials and half-finished products, and workers of K specialties to this end within a period of time under consideration (planning period). Let

a_{ij} be the time of processing a unit of product j on equipment of group i, $i \in \overline{1,n}$, $j \in \overline{1,m}$,
$\tilde{b}_i$ be the time that the equipment of group i can work within the planning period, $i \in \overline{1,n}$,