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Optimal risk management in defined benefit stochastic pension funds☆

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Abstract

We consider a continuous time dynamic pension funding model in a defined benefit plan of an employment system. The benefits liabilities are random, given by a geometric Brownian process. Three different situations are studied regarding the investment decisions taken by the sponsoring employer: in the first, the fund is invested at a constant, risk-free rate of interest; in the second, the promoter invests in a portfolio with n risky assets and a risk-free security; finally, it is supposed that the rate of return is stochastic. Modelling the preferences of the manager such that the main objective is to minimize both the contribution rate risk and the solvency risk, we study cases where the optimal behavior leads to a spread method of funding.

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1. Introduction

The dynamically optimal management of defined benefit pension plans has considerable economic interest, due to the great importance that the world of pensions provision has acquired in financial markets.

In this paper we analyze the optimal contribution rate and asset allocation decisions for a manager of a defined benefit pension plan who wishes to keep the fund as close as possible to prescribed targets. These targets are fixed by actuarial cost methods, allowing an ideal contribution rate—the normal cost—and an ideal fund level—the actuarial liability—to be defined. These ideal levels guarantee the benefits promised to members incorporated to the pension plan over time. However, the existence of uncertainty in some elements of the plan or in the rate of return of the

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assets of the fund can cause the evolution of the plan to be quite different from the initially designed valuation. Hence the contribution rate must be the normal cost modified by a suitable supplementary cost reflecting the disturbances. The supplementary cost is chosen by the promoter with the aim of bringing the expected value of the unfunded actuarial liability to zero. The instruments used by the manager to reach the objectives are the contribution rate and investment earnings.

Along the lines of Haberman (1993), Haberman and Sung (1994), Haberman (1997), Josa-Fombellida and Rincón-Zapatero (2001), we suppose that the aim of the controller is to minimize a combination of the *contribution rate risk* and the *solvency risk*. The former represents the size of deviation of contributions from the normal cost and is related with the stability of the plan. The latter risk is an indicator of the plan's safety, measuring deviations of the fund from the actuarial liability. The model is considered on an unbounded horizon and with a positive discount rate, meaning that the sponsor is more worried about the short-term than with the long run behavior of the fund.

Josa-Fombellida and Rincón-Zapatero (2001) considered the case of a constant value for the benefits, in a context closely related with the framework pioneered by Merton (1971) for optimal consumption and portfolio selection. That paper found that the optimal behavior of the controller leads to a spread method of funding if the technical rate of actualization of the actuarial liability equals the risk-free rate of return. With a spread method the supplementary cost is proportional to the unfunded actuarial liability, in such a way that the corrections made in the rate of contribution to the normal cost are small when the fund is close to the target. Furthermore, it enjoys good stability properties. This is why spread methods have became popular with professionals and institutional agents.

Haberman and Sung (1994) considered a similar model in discrete time, on a finite horizon both in deterministic and stochastic frameworks. These authors do not contemplate investment as an instrumental variable, but all the fund assets are invested at a random rate of return. Another difference with our paper is that they consider constant benefits, whereas in our case we allow stochastic benefits.

Other related papers are O'Brien (1987), Boulier et al. (1995, 1996), Cairns (1995, 1996, 2000), Owadally and Haberman (1999), Vigna and Haberman (2001), Taylor (2002), Chang et al. (2003). Our paper extends the previous analysis incorporating a source of uncertainty in the benefit outgo, supposing that the benefits are given by a geometric Brownian motion. O'Brien (1987) analyzes a stochastic optimal control problem, where the uncertainty in the benefits is modelled in a quite different way. This author makes a linear approximation to the exponential fund model, see Bowers et al. (1986), to retain analytical tractability of the problem. However, no investment decisions are available for the manager, who wishes to maintain a constant fund ratio (with respect to the actuarial liability), and penalizes fluctuations of the contribution rate from zero.

Our paper contemplates three different situations which are studied regarding the investment decisions taken by the sponsoring employer: (i) in the first, the fund is invested at a constant rate of interest; (ii) the promoter invests in a portfolio with n risky assets and a risk-free security; (iii) finally, it is supposed that the rate of return is stochastic. Note that the benefits are a non-tradable process, hence the market is incomplete and, furthermore, we also consider the existence of correlation between the sources of uncertainty in the benefits and in the asset returns. However, the consideration of the benefits as geometric Brownian motion is fundamental to our approach. The problem is solved under particular assumptions-depending on the scenario-concerning the technical rate of actualization and the evolution of the liabilities. These hypotheses are motivated by our intention to show that spread methods lead to a minimization of risk in pension funding, even in a stochastic environment.

An outline of the paper and a summary of the main results are given next. Section 2 provides the definitions of the main elements incorporated in a defined benefit pension plan of an employment system. The actuarial functions, necessary for the valuation of the plan are also introduced, and we prove that when the benefits are given by a geometric Brownian motion, the related actuarial functions are processes of the same type.

Section 3 considers that the fund is invested at a constant and deterministic rate of interest. We find that if the rate of return of the bond is chosen as the valuation rate, then the optimal management of the fund is a spread method. Section 4 presents a model with $n+1$ financial assets, where one of them is a bond. The controller can buy and sell the assets without limitations, with the possibility of selling short and borrowing at the riskless rate of interest. The